ENT

THERMAL ANALYSIS OF LIQUID METAL HEAT EXCHANGERS

By

RAJENDRA SHRIVASTAV

NUCLEAR ENGIN
NDIAN INSTIT

THERMAL ANALYSIS OF LIQUID METAL HEAT EXCHANGERS

A Thesis Submitted
In Partial Fulfilment of the Requirements
for the degree of
MASTER OF TECHNOLOGY

By
RAJENDRA SHRIVASTAV

to the
NUCLEAR ENGINEERING AND TECHNOLOGY PROGRAMME
INDIAN INSTITUTE OF TECHNOLOGY, KANPUR
JUNE, 1977

CERTIFICATE

Certified that the present work 'Thermal Analysis of Liquid Metal Heat Exchanger' by R Shrivastava has been carried out under our supervision and has not been submitted elsewhere for the award of a degree

J SRINIVASAN
Department of Mech Engg
I I T Kanpur

K SRI RAM
Department of Mech Engg
I I T Kanpur

NETP-1977-M-SRI-THE

ACKNOWLEDGEMENT

I thank Dr K Sri Ram and Dr J Srinivasan for their guidance and help

I have all praise for Dr J Srinivasan His indefatigable and deep interest was more than I had in my thesis work

I thank J K Misra for typing my manuscript and Buddhiram for cyclostyling the stencils

Rajendra Shrivastava

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

NOMENCLATURE

Symbols

A_n	Expansion coefficient associated with n-th negative eigenvalue
A_1	Surface area of tube
A_2	Surface area of the Annulus
A_{c1}	Cross-sectional area of the tube
A_{c2}	Cross-sectional area of the annulus
$B_i\{\ \}$	Bulk operator in region i
$\hat{C}_n$	Expansion coefficient associated with n-th positive or negative eigenvalue
C_n	Expansion coefficient associated with n-th positive eigenvalue
c_i	Specific heat of fluid in region i
C_{min}	Minimum heat capacity rate (W/^oC)
D_i	Hydraulic diameter of region i
$\hat{E}_{i,n}(\hat{r}_i)$	Eigenfunction associated with n-th positive or negative eigenvalue
$E_{i,n}(\hat{r}_i)$	Normalized Eigenfunction associated with n-th positive eigenvalue
$E^*_{i,n}(\hat{r}_i)$	Normalized eigenfunction associated with n-th negative eigenvalue
F, F^*	Auxiliary functions
$f_i(\hat{r}_i)$	Total fluid conductivity relative to molecular fluid conductivity in region i.

G, G^*	Auxiliary functions
g	A characteristic function used in presentation of two region Sturum - Liouville problem
$g_1(\hat{r}_1)$	Dimensionless local fluid velocity
$g_1(\bar{x}_1)$	Dimensionless function for large Peclet number approximation
H	Ratio of heat capacity flow rates in region 2 and 1, c_2W_2/c_1W_1
I_o, I_1	Modified Bessel functions of first kind
J_o, J_1	Bessel functions of first kind
K	Relative thermal resistance of fluid, $k_1(1-R)/(k_2\ R)$
K_o, K_1	Modified Bessel functions of second kind
K_w	Relative thermal resistance of wall, $k_1 \ln(r_{21}/r_{22})/k_w$
k	A characteristic function used in presentation of two region Sturm-Liouville problem
k_i	Thermal conductivity of fluid flowing in region i
k_i^+	Approximation associated with large Peclet number
k	Thermal conductivity of tube metal.
L	Heat exchanger length
$\hat{L}$	Non-dimensional heat exchanger length

$\bar{L}$	Nondimensional heat exchanger length associated with large ~~p~~Peclet number approximation
N_n	Normalizing factor associated with n-th positive eigenvalue
N_n^*	Normalizing factor associated with n-th negative eigenvalue
Nu	Nusselt number
$[Nu]_{PF}$	Nusselt number for plug flow
p	Ratio of outer radius of annulus to outer radius of tube (r_{22}/r_{21})
$p_i(\hat{r}_i)$	Outlet temperature distribution of fluid in region i
Pe_i	Peclet number in region i
$Q_{n,k}$	Coefficient of expansion coefficient, in n-th row and k-th column
R	Annulus radius ratio, r_{12}/r_{21}.
Re_i	Reynolds number in region i
$\hat{r}_i$	Radial space variable in region i
r_i	Nondimensional radial space variable in region i
r_{12}	Inner radius of tube
r_{21}	Outer radius of tube or inner radius of annular space.
r_{22}	Outer radius of annular space

St_i	Stanton number in region i
$T_i(r_i, z)$	Local temperature in region i
T_{10}	Temperature of fluid entering tube (region 1)
T_{2L}	Temperature of fluid entering annulus (region 2)
$\hat{T}_i(\hat{r}_i, \hat{z})$	Nondimensional temperature distribution in region i
$\tilde{T}_i(\tilde{r}_i, \tilde{z})$	Nondimensional temperature distribution associated with large ~~p~~Peclet number approximation, in region i.
$u_i(r_i)$	Local velocity in region i
$u_i(\hat{r}_i)$	Nondimensional local velocity in region i
W_i	Mass flow rate in region i
x	Dependent variable in two region Sturm-Liouville problem
$\tilde{x}_i(\hat{r}_i)$	Nondimensional radial space variable associated with large Peclet number approximation for plug flow, in region i
Y_0, Y_1	Bessel functions of second kind
y	Characteristic function used in presentation of two region Sturm-Liouville problem
z	Heat exchange length variable
$\hat{z}$	Nondimensional heat exchanger length

$\bar{z}$ Nondimensional heat exchanger length associated with large Peclet number approximation for plug flow

Greek Letters

α_1	Thermal diffusivity in region 1
β_n^2	Absolute value of n-th negative eigenvalue.
ε	Heat exchanger efficiency
ε_{H_1}	Eddy diffusivity of heat in region 1
$\theta(z)$	Function associated with nondimensional length in energy equation solution
λ_n^2	n-th positive eigenvalue
σ	R/(1-R)
$\phi(\lambda_n)$	Symbol denoting eigenvalue equation associated with positive eigenvalues
$\phi^*(\beta_n)$	Symbol denoting eigenvalue equation associated with negative eigenvalues
ω^2	HKR/(1+R)

Abbreviations

FBR	Fast Breeder Reactor
FBTR	Fast Breeder Test Reactor
IHX	Intermediate Heat Exchanger
NTU	Number of Transfer Units
UHF	Uniform Heat Flux
SG	Steam Generator

ABSTRACT

A detailed analysis of temperature variation within the sodium-sodium intermediate heat exchanger of the fast breeder test reactor at Kalpakkam has been performed This analysis avoids the use of empirically determined heat transfer coefficients since it may cause large errors in the case of liquid metals The analysis involves a two-region Sturm-Liouville problem with both negative and positive eigenvalues The results of the present analysis predict the efficiency of the heat exchanger to be six percent less than that predicted by the approximate method using Nusselt numbers for constant heat flux. It is shown that the approximate method can lead to substantial errors if the ratio of heat capacity rates of the two streams is around 2 (or 0 5) and the heat exchanger is not long

CHAPTER 1

INTRODUCTION

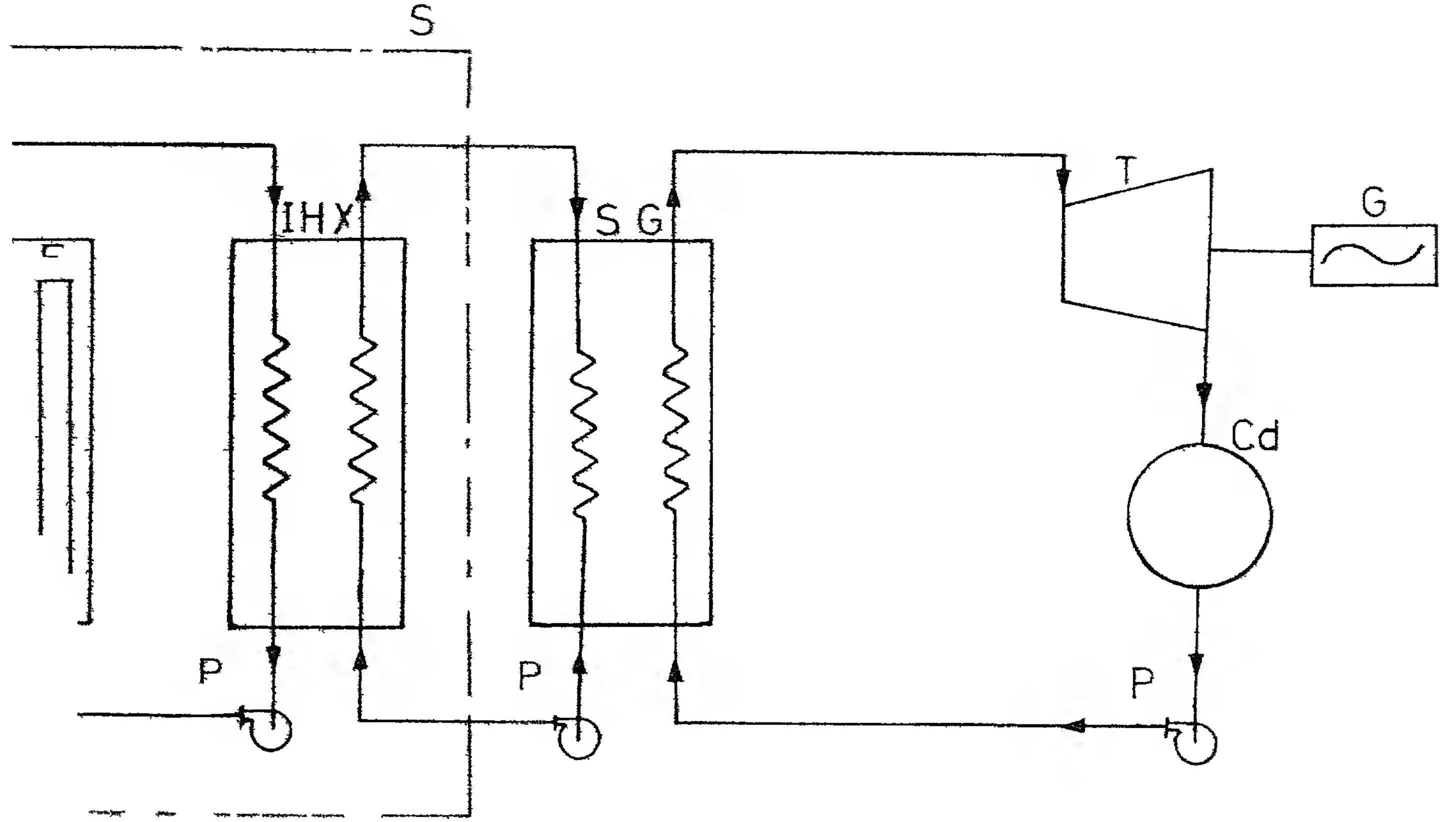
[1 1] INTRODUCTION

Nuclear power competes commercially at present with any other conventional source of power like thermal power or hydel power There are more than hundred nuclear power plants in commercial operation, at present, all over the world

Nuclear power is produced by controlled fission of Nuclear fuel like Uranium-235 or Plutonium-239 Each fission releases energy which is responsible ultimately for generating power This chain reaction is carried in a reactor core The core is normally composed of fuel, moderator, coolant, control devices and structural material Coolant which carries heat from hot fuel elements may also act as a moderator in the core If moderator is removed from the core and fission chain reaction is maintained without slowing down the neutrons, then the reactor is called fast reactor Its counterpart with moderator is known as thermal reactor

Transfer of heat to the coolant inside core, is mainly by conduction and convection This coolant is pumped to an

FIG. 11



TUEL
COOLANT
CONDENSER
INTERMEDIATE
HEAT EXCHANGER
GENERATOR

P PUMP
RC REACTOR CORE
S SHELDING
SG STEAM GENERATOR
T TURBINE

intermediate heat exchanger, (IHX) A secondary fluid picks up heat in IHX and goes to steam generator (SG) Steam produced in SG runs steam turbine which is coupled to electric generator Therefore the major difference in conventional and nuclear power plant lies in the mode of generation of primary heat

[1 2] FAST REACTORS

In fast reactors the neutrons are not deliberately slowed down by a moderator but only by inelastic collisions of neutrons with fuel and other reactor material nuclei, Fig (1 1 Fast reactors have very high power densities. A typical fast reactor has power density about 40 times that of heavy water moderated natural uranium thermal reactor

Liquid metals are the only possible coolants for fast reactors, because of high power generation per unit volume, within the core. Liquid metals have excellent heat transfer characteristics which results in low hot spot factors, i e , less local temperature variations Use of liquid metals give high thermal efficiency.

There is very high neutron flux inside reactor core This results in high level of induced radioactivity in the reactor materials and liquid metal coolant Coolant is circulated in the heat exchanger and therefore there is a chance of leakage of radioactive coolant to the other side of the heat exchanger

If on the other side, the working fluid is steam then the leakage of radioactive coolant to the steam side may expose the working personnel (dealing with the turbine or condenser) to harmful radiation

Radio active sodium has a long half life of 15 hrs and it emits high energy gamma rays Hence leakage of sodium to steam cycle is dangerous This is avoided by employing an intermediate heat exchanger (IHX), in which liquid sodium flows on both the sides This reduces the possibility of radioactive contamination of steam because now the steam is generated in steam generator, where the secondary sodium (not active) is flowing on the other side

[1 3] FAST BREEDER REACTORS (FBR)

Interest in fast reactors stem from the fact that they offer a singular possibility of converting fertile material to fissile material Fast reactors can have ratio of conversion of fissile material generated (from fertile material) to fissile material consumed, equal or greater than one Such a reactor is termed as fast breeder reactor (FBR)

FBR thus offersthe possibility of producing power and nuclear fuel simultaneously This can be achieved economically only when many of the technological hurdles are overcome

[1 4] LIQUID METALS

Fast reactors do not have moderators and therefore it must be ensured that coolant does not act as moderator Hence the coolant nucleus must have small cross-section for interaction with neutrons This coolant has to satisfy stringent requirements as regards its thermal properties As mentioned earlier the rate of heat generation inside fast reactor core is very high Therefore coolant must have high thermal conductivity, low vapour pressure, high specific heat and high volumetric heat capacity These properties will enable it to transport heat from core rapidly with less pumping power and smaller area of contact For reactors with high thermal fluxes and operating at high temperatures, liquid metals have special interest as coolants Liquid metals have excellent thermal properties and they fulfil the above mentioned requirements quite well

Amongst various liquid metals sodium is the most suitable coolant for a reactor operating at high temperatures, particularly for fast reactors where moderating (low mass number) materials cannot be used In fact sodium and sodium potassium alloys have been successfully used as coolants in both fast and thermal reactors. They are also used as secondary coolants in intermediate heat exchangers, in nuclear reactor power plants

[1 5] LIQUID METAL HEAT EXCHANGERS

Research in liquid metal heat transfer area is going on for last three decades Most of it was devoted to developing correlations for use in engineering design ~~The~~ accurate and reliable prediction ~~were~~ was not considered important till recently The main reason for this was relatively poor thermal properties of heat exchanger wall material This resulted in the temperature differences across the wall thickness an order of magnitude higher than temperature differences associated with convection to liquid metals Therefore, fairly large inaccuracies (say $\pm$ 30 percent) of liquid metal heat transfer coefficients did not greatly affect the overall thermal design But for liquid metal heat exchangers used in fast breeder reactors a very accurate thermal design is required on account of safety considerations

Liquid metals have certain peculiar properties In liquid metal heat exchangers the use of conventional heat transfer coefficient concept is not valid, hence the existing methods for nonmetallic fluids are not directly applicable to liquid metals

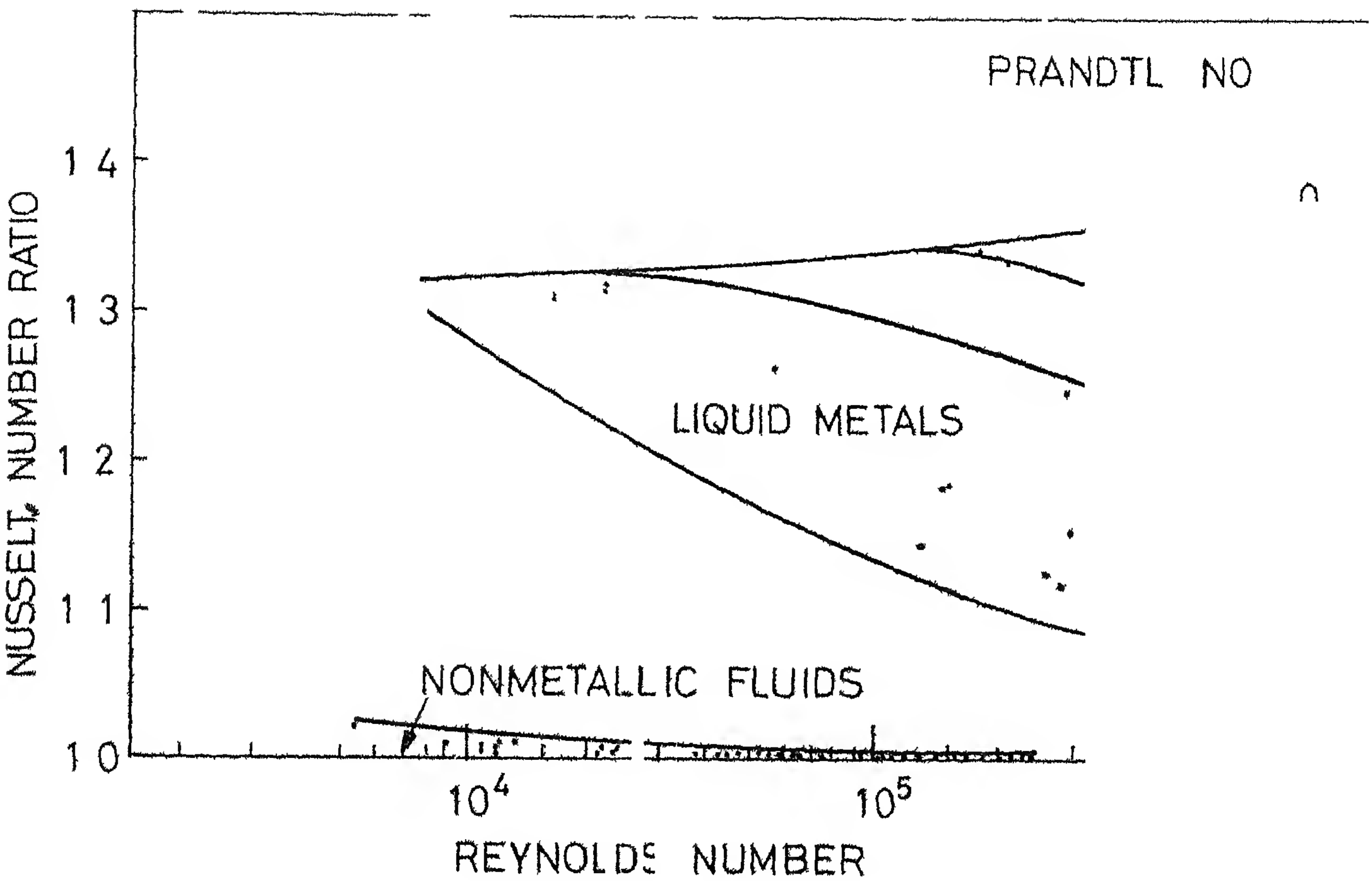
Liquid metals have small viscosities Therefore their application as coolant or heat exchanger fluid, e g., in nuclear reactors, almost always involves large Reynold no and consequently turbulent flow, Thermal conductivities

of liquid metals are very high, hence molecular conduction remains significant throughout the cross-section of flow channel or duct geometry Therefore temperature gradients are not localized near the wall as it is the case with nonmetallic fluids This is a characteristic difference between turbulent convection in liquid metals and nonmetals

Prandtl numbers for liquid metals are very small Therefore, Peclet numbers associated with even turbulent flow may be as low as 50, with nonmetallic fluids it is rarely so Low Peclet number associated with high thermal conductivity results in axial heat conduction Millsaps and Pohlhausen (1956) and Singh (1957) have shown that upto a Peclet number of 100 axial conduction is significant But error due to neglect of axial conduction even at a low Peclet number may not be large Schneider (1956) determined that for Peclet number of 10 and for length to diameter ratio of unity, the neglect of axial conduction gave error of only 4 percent

Liquid metal Nusselt numbers are very sensitive to boundary conditions Sleicher and Tribus (1957) have computed the ratio of Nusselt numbers for uniform heat flux to that for uniform wall temperature for different fluids as a function of Reynolds number This is shown

FIG 1 2



Effect of ɔounc conditions on fully developec Nuss numbers; ratio of values fo unif orm c ter
(W

in Fig (1 2) From this figure we conclude that non metallic fluids are insensitive to the type of boundary conditions specified Metallic fluids on the other hand are very sensitive to this and variation in Nusselt numbers for two different cases may be as high as 40 percent

In liquid metal heat exchangers the heat flux is rarely uniform on the other hand boundary conditions which specify the wall temperatures are not often accurate description of actual situations of practical interest in liquid metal heat transfer In certain simple condensers or boilers, wall temperature boundary conditions can serve as reasonable approximations, but with liquid metals this approximation may be quite inaccurate. For example, because of excellent heat transfer properties of liquid metals, conduction through the condenser or boiler walls will be much more important than with nonmetallic fluids

With heat transfer coefficients sensitive to boundary conditions and large number of boundary conditions that are of practical interest, most of the convenience of traditional heat transfer coefficient concept is lost Hence one has to start from the statement of basic energy equation for the given configuration coupled with the careful statement of boundary conditions. Assumption of uniform heat flux of uniform wall temperature may lead to significant inaccuracies

The thermal modelling of IHX of -FBTR at Kalpakkam is considered in this thesis. The temperature profile within the heat exchanger is obtained by solving the basic differential equations of heat transfer The use of the concept of heat transfer coefficient is scrupulously avoided This is because it has been shown by Stein (1966) and others that the use of heat transfer coefficient concept can lead to great errors in the prediction of temperature profiles, in liquid metal heat exchangers

CHAPTER 2

MODELLING OF INTERMEDIATE HEAT EXCHANGER AT KALPAKKAM

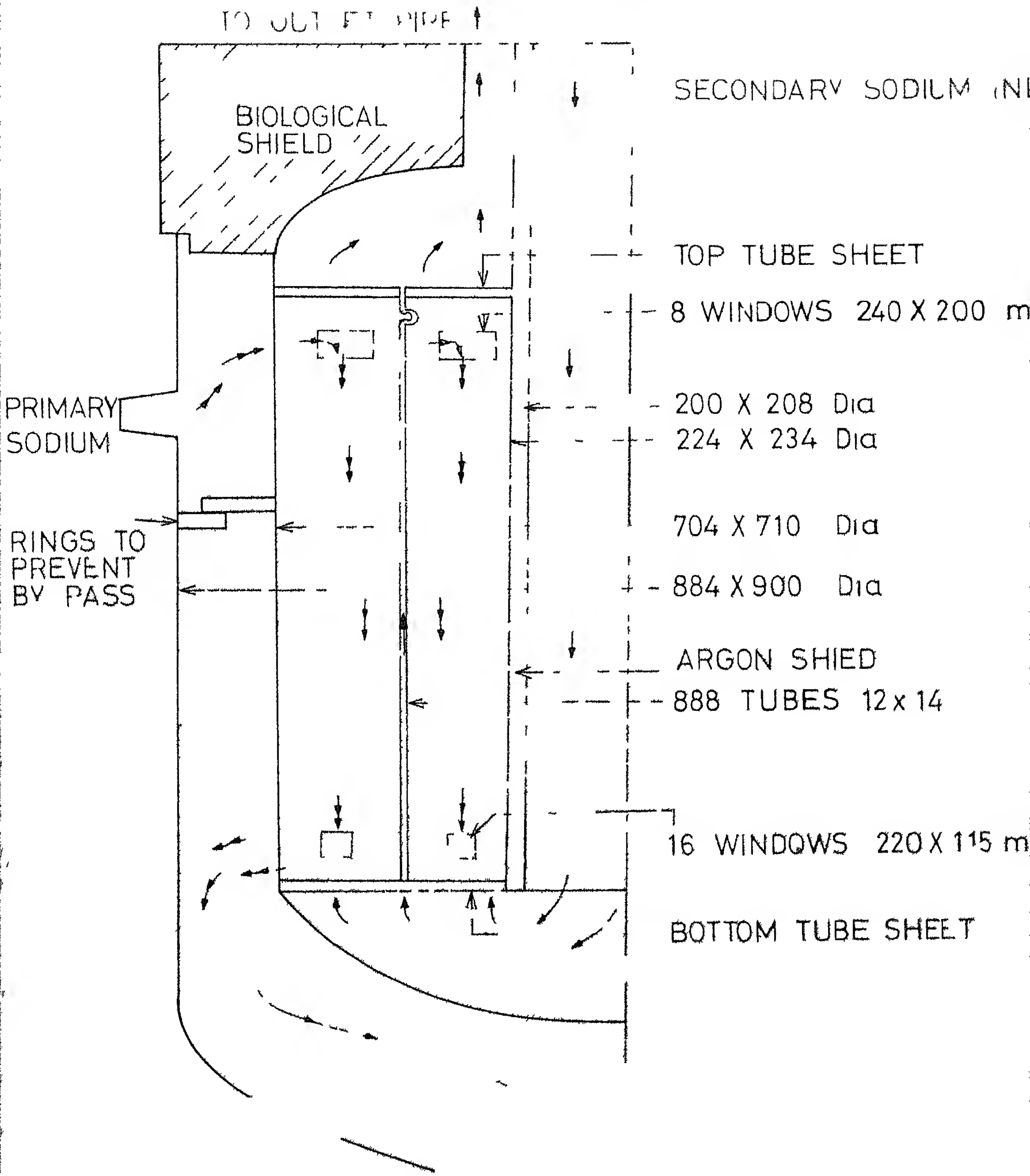
[2.1] INTRODUCTION

A fast breeder reactor (FBR) power plant is coming up at Kalpakkam, near Madras This plant is expected to produce power, commercially, from early ninties Its design is based on the French FBR called RAPSODIE A detailed description of this reactor system is given by Yevick (1966) This FBR can be used to breed Plutonium - 239, a fissile material, using Uranium - 238 as its parent fertile material

A prototype of the commercial reactor is likely to come into - operation in early eigthies Its designed capacity is 45 MW th A detailed description of its intermediate heat exchanger is given here

The prototype is called fast breeder test reactor (FBTR) FBTR is a sodium cooled reactor The heat produced at the reactor core is removed by circulating sodium through the core The primary heated sodium then transfers heat to a secondary sodium at IHX. This secondary sodium then heats up the water in steam generator, for running steam turbine to produce electric power.

FIG 21


TO OUTLET PIPE
SECONDARY SODIUM INL
BIOLOGICAL SHIELD
TOP TUBE SHEET
8 WINDOWS 240 X 200 m
PRIMARY SODIUM
200 X 208 Dia
224 X 234 Dia
RINGS TO PREVENT BY PASS
704 X 710 Dia
884 X 900 Dia
ARGON SHIED
888 TUBES 12 x 14
16 WINDOWS 220 X 115 m
BOTTOM TUBE SHEET

There are two IHX in FBTR and they share the load equally viz 22 5 MW th Their normal operating condition is given in Table 2 1

Table 2 1 Normal Operating Condition (For single IHX)

Power exchanged	22 5 MW th	
Primary fluid	Sodium (active)	
Primary fluid flow rate	125 kg/sec	
Primary inlet temp	520°C	
Primary outlet temp	380°C	
Secondary fluid	Sodium (not active)	
Secondary inlet temp	283°C	
Secondary outlet temp	514°C	
Pressure (inlet to unit)	5 03 bars abs.	Primary
Pressure (outlet to unit)	4 81 bars abs	fluid
Secondary flow rate	74 kg/sec	

[2 2] INTERMEDIATE HEAT EXCHANGER

There are two IHX They consist of a fixed shell, a removable portion having the tube bundle and biological shield and other provisions for IHX instrumentation and control, Fig (2.1)

a) Primary Side

The primary sodium system components have been designed in such a way that the main equipments, e g , heat exchanger pumps could be removed without disturbing the primary circuit Hence the components have a fixed tank connected with the primary circuit in which free sodium level is maintained with Argon gas cover (See Fig (2 1)) The removable portion is introduced into this from the top The fixed shell of the IHX is a stainless steel vessel The vessel has a primary sodium inlet nozzle on its vertical portion and outlet nozzle on the dished nozzle enclosure at the bottom The vessel has double envelop till the height upto which sodium is likely to be present Hot nitrogen could be circulated through the nozzle arrangement around annulus through the shell and its envelop, to preheat equipment before charging with sodium

Electrical resistance type heaters are mounted on double envelope of this vessel for heating up this equipment during the initial testing of the secondary loop

b) Secondary Side

There are 888 stainless steel tubes of 12/14 mm in the tube bundle welded to tube sheets at each end Tubes are arranged over 12 pitch circles around a central pipe of

diameter 224/234 mm (see Table 2 2) This central pipe is also welded to the tube sheets at either end The secondary sodium inlet pipe of diameter 200/208 mm passes inside this pipe and is welded only to bottom sheet The annulus between two pipes is kept under argon atmosphere The differential expansion between the central pipe and the tubes and also among tubes is accommodated by giving a wave shape (expansion bends) to the tubes at the top

There is a vertical baffle around the tube bundle There are inlet and outlet windows for primary sodium flow, in this vertical shell The shell is free to move vertically during thermal expansion By means of a ring provided around this shell, it rests over another ring of fixed shell, thereby avoiding by pass of the heat exchanger by primary sodium flow There are also a few vertical and circumferential baffles on the outerside of the vertical shell arranged around the top inlet windows to achieve uniform flow distributions around windows and also flow without any gas entrainment problem

Table 2 2 Arrangement of tubes

Row	No. of tubes	Row	No. of tubes
1	41	7	77
2	47	8	83
3	53	9	89
4	59	10	95
5	65	11	101
6	71	12	107

Total No of Tubes 888 , Pitch, radial 19 mm

pitch, circumferential 20 mm

There are temporary block devices to prevent incidental movement of this vertical shell during handling of this equipment

The entry header for the secondary tube bundle is formed by a dished end welded to the top tube sheet Secondary sodium enters at the top, flows all the way down through its inlet pipe It collects at the bottom dished end before passing through tubes It leaves IHX at the top

The primary sodium enters through the top of the nozzle of the fixed shell It gets distributed around in annulus It rises above the rings described earlier and enters the shell through windows at the top It flows downwards from there around the vertical tubes which carry secondary sodium It finally flows out through the outlet steam nozzle The top dished end collectors and outlet pipe also have electrical resistance heater provided for preheating

c) <u>Biological Shield Plug</u>

The plug and the integral part of the removable portion of IHX form the closure of this equipment at the top It rests over the top flange of this fixed shell and the sealing towards the outside atmosphere is assured by means of two 'O' rings The parts of the heat exchangers which come in contact with sodium are made of Austenitic stainless steel AISI 316

[2 3] MODELLING

The IHX is a shell and tube type heat exchanger It is a single pass counter-flow heat exchanger Secondary sodium flows all the way down to the dished end and then rises to top exit There is practically no heat transfer from or to the secondary fluid during its downward flow because of argon shield around it Primary fluid enters the annulus around removable shell at a height such that it is close to top entrance windows Therefore this heat exchanger can be approximately considered to be a single pass counter flow, shell and tube type heat exchanger Hot fluid (primary active sodium) flowing outside tubes and secondary fluid (sodium) flowing inside the tubes

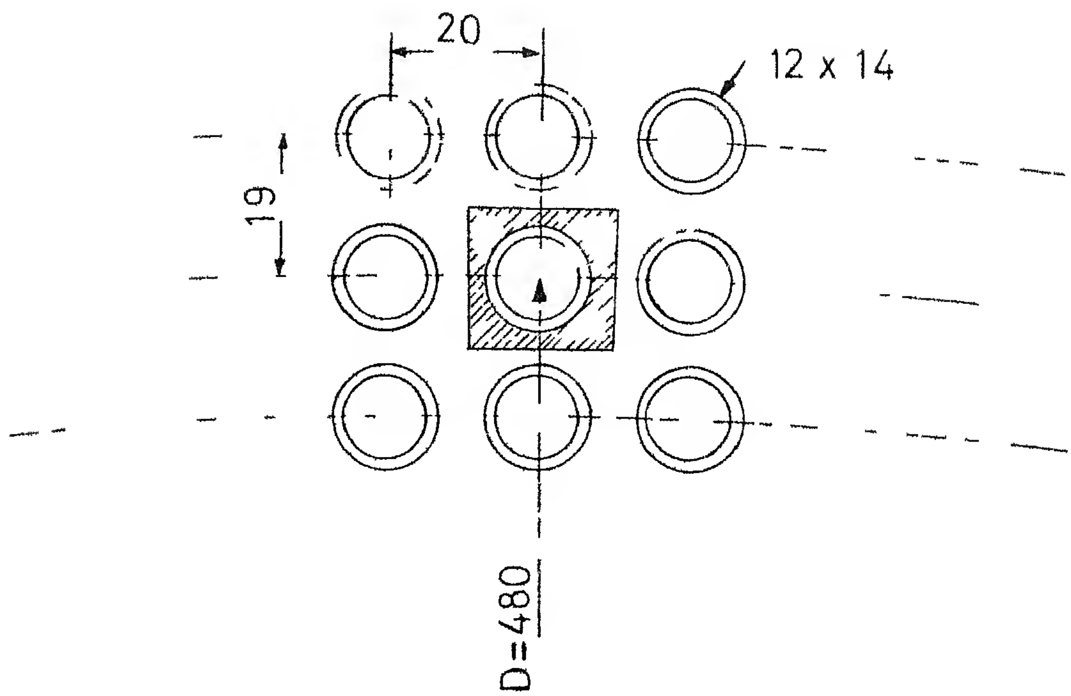
The geometrical configuration of the heat exchanger tubes is such that the problem is one of heat transfer across tube bundles for in line flow of fluid This problem has been studied by many investigators, employing different methods

a) Equivalent annulus approach was adopted by Dwyer and Tu (1960), Friedland and Bonilla (1961) and others

b) Graphical and lumped parameter methods were used by Deissler and Taylor (1957), Dwyer (1966) and others.

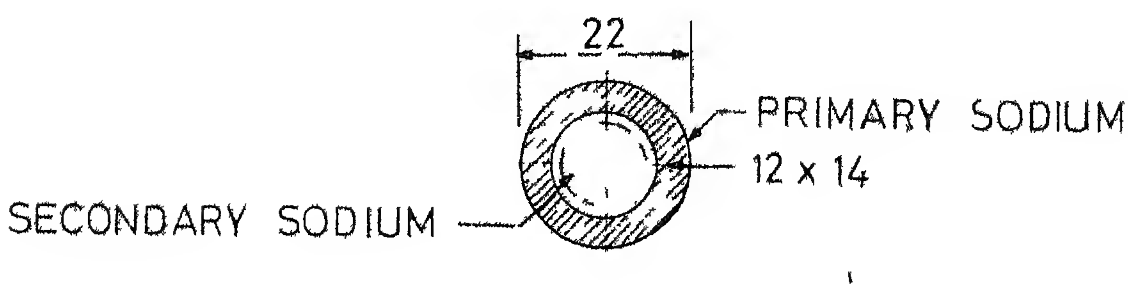
FIG 2 2 (a)

All dimensions are in mm



Characteristic flow region

FIG 2 2 (b)



Ch[illegible]eristic [illegible] n converted to [illegible] valent

c) Finite difference method was employed by Bender and Magee (1969) and Oberjohn (1970)

[2 4] EQUIVALENT ANNULUS APPROACH

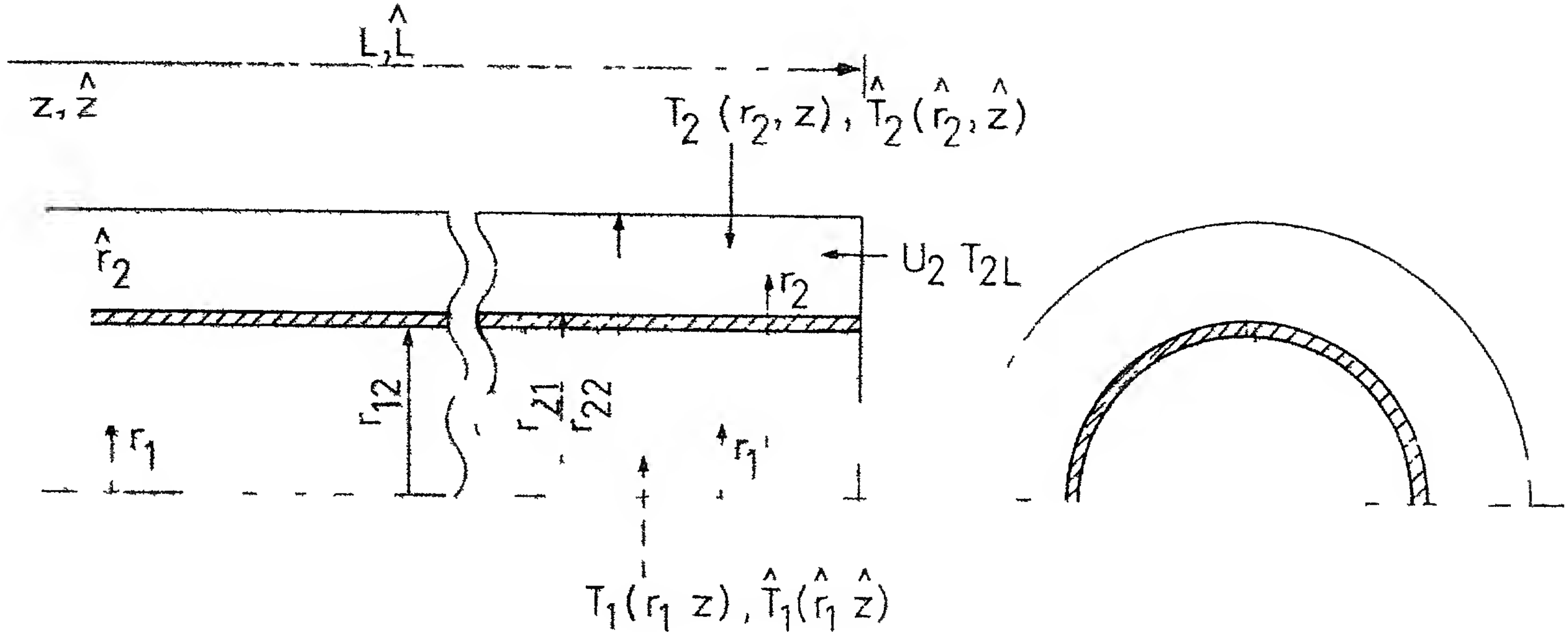
In a shell and tube type heat exchanger, the tube centres usually make a finite number of equilateral arrays, across the heat exchanger cross-section They are individually called unit cell or lattice The cell may be a regular polygon e g it is rectangular in IHX considered here, Fig (2 2a) Each unit cell represents typically the thermal behaviour of entire heat exchanger

Once unit cell is recognized, characteristic flow region is determined around each tube, Fig (2 2a) It is also of same geometry as its parent cell geometry The boundaries of the characteristic flow region are considered to be adiabatic If this characteristic flow region is circular, the mathematical analysis becomes very much simplified This is unfortunately not the case with almost all shell and tube type heat exchangers In equivalent annulus approach this characteristic flow region is replaced by an imaginary circular characteristic flow region around the tube, Fig (2 2b) Outer boundary of the annulus is considered insulated The annular flow area is taken equal to the flow area of the characteristic flow region in Fig (2 2a)

Nijsing and Eifler (1973) compared results of different approaches, mentioned above, along with the approximate analytical method they used. They have shown that when in equivalent annulus approach the ratio of outer radius of annulus to outer radius of tube i.e., $p \geq 1.15$, the results obtained using equivalent annulus approach do not differ significantly from that obtained by any other method.

The Kalpakkam IHX has $p = 1.57$, therefore we adopt equivalent annulus approach. The mathematical simplification offered using this method outweigh the insignificant errors contained in the final results.

FIG 3.1



ıe double pipe countercurrent flow heat exchanger

CHAPTER 3

MATHEMATICAL ANALYSIS

[3 1] INTRODUCTION

Employing equivalent annulus approach the shell and tube type IHX at Kalpakkam is converted to an equivalent IHX where each tube is surrounded by an imaginary annulus Outer boundary of the annulus is adiabatic Hence the problem to be solved is counter-current, double pipe heat exchanger where on both sides liquid metal (sodium) is flowing A line diagram, Fig (3 1), represents typical annulus where in regions 1 and 2 secondary and primary sodium are flowing Brown (1968) presented mathematical analysis of such a system He basically extended the work of Stein (1966). Stein discussed the problem in generality considering symmetrical and asymmetrical ducts, co-current and counter-current flows, with constant and varying temperature and heat flux conditions We will use his procedure to formulate the problem Final results are obtained using Brown's modifications which simplify the numerical calculations

[3 2] ASSUMPTIONS

The general equation for energy balance for double pipe heat exchanger precludes any possibility of solving it by analytical means Following assumptions are made to simplify the equation

i) The fluids enter the heat exchanger at uniform temperature

ii) Physical properties are independent of temperature

iii) Frictional heating is negligible

iv) Axial heat conduction within the heat exchanger walls is negligible

v) Axial heat conduction within the heat exchanger fluids is negligible

vi) Velocity distribution of the fluids is independent of axial position (i e , fully developed) and is known function of radial variable

vii) The heat exchanger is in steady state operation

The first four assumptions are always almost attainable in a physical system Fifth assumption for liquid metals is valid if Peclet number is greater than 50 The sixth assumption is realized in liquid metals because of their small viscosities They attain fully developed velocity profiles very early compared to thermal fully developed profiles For large Reynolds number (Re No $> 10^4$) the velocity profile is nearly flat even close to entrance region

[3 3] DIMENSIONAL FORMULATION

The general energy equations for tube and annulus, with the stated assumptions can be written as follows

Tube

$$\frac{1}{r_1}\frac{\partial}{\partial r_1}\left[\left(1+\frac{\varepsilon_{H1}}{\alpha_1}\right) r_1 \frac{\partial T_1(r_1,z)}{\partial r_1}\right]$$

$$= \frac{1}{\alpha_1}\left(u_1(r_1)\right)\frac{\partial T_1(r_1,z)}{\partial z} \qquad (3\ 1)$$

Annulus

$$\frac{1}{r_2}\frac{\partial}{\partial r_2}\left[\left(1+\frac{\varepsilon_{H2}}{\alpha_2}\right) r_2 \frac{\partial T_2(r_2,z)}{\partial r_2}\right]$$

$$= \frac{1}{\alpha_2}\left(-u_2(r_2)\right)\frac{\partial T_2(r_2,z)}{\partial z} \qquad (3\ 2)$$

where,

ε_{Hi} = Eddy diffusivity of heat in region i

α_i = Thermal diffusivity of fluid in region i

i = 1,2

The above equations are valid both for laminar and turbulent flow

Boundary conditions for these equations are as follows

$$T_1(r_1, 0) = T_{10} \text{ (constant)} \qquad (3\ 3)$$

$$T_2(r_2, L) = T_{2L} \text{ (constant)} \qquad (3\ 4)$$

Tube centre line, outer annulus,

$$\frac{\partial T_1(0,\ z)}{\partial r_1} = 0\,, \qquad \frac{\partial T_2(1\ z)}{\partial r_2} = 0\,, \tag{3.5}$$

continuity of heat flux across inner wall

$$r_{12}\, k_1 \frac{\partial T_1(r_{12}, z)}{\partial r_1} = r_{21}\, k_2 \frac{\partial T_2(r_{21}, z)}{\partial r_2} \tag{3 6}$$

conduction of heat across inner wall

$$-r_{12}\, k_1 \frac{\partial T_1(r_{12}, z)}{\partial r_1} = \frac{r_{21}\ k_w}{\ln\,(r_{21}/r_{12})} \left[T_1(r_{12}, z) - T_2(r_{21}, z)\right] \tag{3.7}$$

where,

k_i = thermal conductivity of fluid in region i

i = 1, 2

k_w = thermal conductivity of tube metal

[3 4] DIMENSIONLESS FORMULATION

We introduce following non-dimensional variables,

$$\hat{r}_1 = \frac{r_1}{r_{12}} \quad \text{where} \quad 0 \le \hat{r}_1 \le 1 \tag{3 8}$$

$$\hat{r}_2 = \frac{r_2 - r_{21}}{r_{22} - r_{21}} \quad \text{where} \quad 0 \le \hat{r}_2 \le 1 \tag{3 9}$$

$$\hat{z} = \frac{4z}{Pe_1(2r_{12})} \quad \text{where} \quad 0 \le \hat{z} \le \hat{L} \tag{3 10}$$

$$\hat{T}_i(\hat{r}_i,\hat{z}) = \frac{T_i(r_i,z)-T_{10}}{T_{2L}-T_{10}} \text{ where } 0 \leq \hat{T}_i \leq 1$$

$$i = 1,2 \qquad (3\ 11)$$

and Pe_i = Peclet number of fluid flowing in region i

Substituting these parameters in Eq (3.1) and (3 2), the governing energy equations become,

<u>Tube</u>

$$\frac{1}{\hat{r}_1}\frac{\partial}{\partial\hat{r}_1}[f_1(\hat{r}_1)\,\hat{r}_1\,\frac{\partial\hat{T}_1(\hat{r}_1,\hat{z})}{\partial\hat{r}_1}] = g_1(\hat{r}_1)\,\frac{\partial\hat{T}_1(\hat{r}_1,\hat{z})}{\partial\hat{z}} \qquad (3\ 12)$$

<u>Annulus</u>

$$\frac{1}{\hat{r}_2+\sigma}\frac{\partial}{\partial\hat{r}_2}[f_2(\hat{r}_2)(\hat{r}_2+\sigma)\,\frac{\partial\hat{T}_2(\hat{r}_2,\hat{z})}{\partial\hat{r}_2}]$$

$$= -\,g_2\,(\hat{r}_2)\,\omega^2\,\frac{\partial\hat{T}_2(\hat{r}_2,\hat{z})}{\partial\hat{z}} \qquad (3\ 13)$$

and boundary conditions become

<u>Entrance</u>

$$\hat{T}_1\,(\hat{r}_1,\,0) = 0 \qquad (3\ 14)$$

$$\hat{T}_2\,(\hat{r}_2,\,\hat{L}) = 1 \qquad (3\ 15)$$

<u>Interior</u>

$$\frac{\partial\hat{T}_1(0,\,\hat{z})}{\partial\hat{r}_1} = 0 \qquad (3\ 16)$$

$$K \frac{\partial \hat{T}_1(1, \hat{z})}{\partial \hat{r}_1} = \frac{\partial T_2(0, \hat{z})}{\partial \hat{r}_2} \qquad (3\ 16)$$

$$K_w \frac{\partial \hat{T}_1(1, \hat{z})}{\partial \hat{r}_1} = \hat{T}_2(0, \hat{z}) - \hat{T}_1(1, \hat{z}) \qquad (3\ 18)$$

$$\frac{\partial T_2(1, \hat{z})}{\partial \hat{r}_2} = 0 \qquad (3\ 19)$$

where,

$$f_i(\hat{r}_i) = 1 + \frac{\varepsilon_{Hi}}{\alpha_i} \qquad (3\ 20)$$

$$g_i(\hat{r}_i) = \frac{u_i(\hat{r}_i)}{u_{iav}} \qquad (3\ 21)$$

$u_i(\hat{r}_i)$ = Dimensional velocity of fluid in region i

u_{iav} = Average velocity in region i,

i = 1, 2

$$R = r_{21}/r_{22} \qquad (3\ 22)$$

$$\sigma = R/(1-R) \qquad (3\ 23)$$

$$H = \frac{c_2 W_2}{c_1 W_1} \qquad (3\ 24)$$

c_i = specific heat of fluid flowing in region i

W_i = Mass flow rate in region i

$$K = \frac{k_1}{k_2} \frac{(1-R)}{R} \qquad (3\ 25)$$

$$K_w = \frac{k_1}{k_w} \ln (r_{21}/r_{22}) \tag{3 26}$$

$$\omega^2 = \frac{HKR}{1+R} \tag{3 27}$$

[3 5] SOLUTION OF GOVERNING EQUATIONS

The Eq (3 12) and (3 13) are solved using separation of variables method This leads to the Sturm-Liouville problem for two regions involving discontinuity at interface Solution of such problem involves positive and negative eigenvalues and their corresponding eigenfunctions A detailed mathematical analysis is given in Appendix A, for the solution of the Eq (3 12) and (3 13) with the stated boundary conditions

[3 6] TEMPERATURE DISTRIBUTION

The general solutions for temperature distribution, are as follows

<u>Tube</u>

$$\hat{T}_1(\hat{r}_1,\hat{z}) = C_o + \sum_{n=1}^{\infty} [A_n E^*_{1,n} (\hat{r}_1) e^{-\beta_n^2(\hat{L}-\hat{z})} + C_n E_{1,n} (\hat{r}_1) e^{-\lambda_n^2 \hat{z}}] \tag{3.28}$$

<u>Annulus</u>

$$\hat{T}_2(\hat{r}_2,\hat{z}) = C_o + \sum_{n=1}^{\infty} [A_n E_{2,n} (\hat{r}_2) e^{-\beta_n^2(\hat{L}-\hat{z})} + C_n E_{2,n} (\hat{r}_1) e^{-\lambda_n^2 \hat{z}}] \tag{3 29}$$

where,

λ_n^2 = n-th positive eigenvalue

β_n^2 = Absolute value of the n-th negative eigenvalue,

$E^*_{1,n}(\hat{r}_1)$ = Normalized eigenfunction in region 1 associated with n-th positive eigenvalue

$E_{1,n}(\hat{r}_1)$ = Normalized eigenfunction in region 1 associated with n-th negative eigenvalue

C_n = Expansion coefficient associated with n-th positive eigenvalue

A_n = Expansion coefficient associated with n-th negative eigenvalue

[3.7] EIGENVALUE EQUATIONS

The ~~x~~eigenvalues λ_n^2 and β_n^2 are the squares of the positive roots of the equations,

$$\phi(\lambda_n) = 0 \tag{3 30}$$

$$\phi^*(\beta_n) = 0 \tag{3 31}$$

where,

$$\phi(\lambda_n) = K_W \frac{dG(0, \lambda_n)}{d\hat{r}_2} \frac{dF(1, \lambda_n)}{d\hat{r}_1} + \frac{dG(0, \lambda_n)}{d\hat{r}_2} F(1, \lambda_n)$$

$$- K \frac{dF(1, \lambda_n)}{d\hat{r}_1} G(0, \lambda_n) \tag{3 32}$$

$$\phi^*(\beta_n) = K_W \frac{dG^*(0, \beta_n)}{d\hat{r}_2} \frac{dF^*(1, \beta_n)}{d\hat{r}_1} + \frac{dG^*(0, \beta_n)}{d\hat{r}_2}$$

$$F^*(1, \beta_n) - K \frac{dF^*(1, \beta_n)}{d\hat{r}_1} G^*(0, \beta_n) \quad (3.33)$$

The functions F, F^*, G^* and G are defined by following initial value equations for auxiliary functions

[3.8] AUXILIARY FUNCTIONS

$$\frac{d}{d\hat{r}_1}\left[f_1(\hat{r}_1)\ \hat{r}_1 \frac{dF(\hat{r}_1, \lambda_n)}{d\hat{r}_1}\right] + \lambda_n^2\ \hat{r}_1\ g_1(\hat{r}_1)\ F(\hat{r}_1, \lambda_n) = 0$$

(3.34)

where,

$$\frac{dF(0, \lambda_n)}{d\hat{r}_1} = 0, \quad F(0, \lambda_n) = 1$$

$$\frac{d}{d\hat{r}_1}\left[f_1(\hat{r}_1)\ \hat{r}_1 \frac{dF^*(\hat{r}_1, \beta_n)}{d\hat{r}_1}\right] - \beta_n^2\ \hat{r}_1\ g_1(\hat{r}_1)\ F^*(\hat{r}_1, \beta_n) = 0$$

(3.35)

$$\frac{dF^*(0, \beta_n)}{d\hat{r}_1} = 0; \quad F^*(0, \beta_n) = 1$$

$$\frac{d}{d\hat{r}_2}\left[f_2(\hat{r}_2)\ (\hat{r}_2+\sigma) \frac{dG(\hat{r}_2, \lambda_n)}{dr_2}\right] - \lambda_n^2\ \omega^2\ g_2(\hat{r}_2)$$

$$(\hat{r}_2 + \sigma)\ G(\hat{r}_2, \lambda_n) = 0 \quad (3.36)$$

$$\frac{dG(1,\ \lambda_n)}{d\hat{r}_2} = 0\ , \quad G(1,\ \lambda_n) = 1$$

$$\frac{d}{d\hat{r}_2}\left[f_2(\hat{r}_2)\ (\hat{r}_2 + \sigma)\ \frac{dG^*(\hat{r}_2,\ \beta_n)}{d\hat{r}_2}\right] + \beta_n{}^2\ \omega^2\ g_2(\hat{r}_2)$$

$$(\hat{r}_2 + \sigma)\ G^*\ (\hat{r}_2,\ \beta_n) = 0 \qquad (3.37)$$

$$\frac{dG^*(1,\ \beta_n)}{d\hat{r}_2} = 0\ , \quad G^*\ (1,\ \beta_n) = 1$$

[3.9] EIGENFUNCTIONS

The normalized eigenfunctions are given by the following equations,

$$E_{1,n}\ (\hat{r}_1) = \left[\ \frac{dG(0,\ \lambda_n)}{d\hat{r}_2}\ F\ (\hat{r}_1,\ \lambda_n)\right] / \sqrt{|N_n|} \qquad (3.37)$$

$$E^*_{1,n}\ (\hat{r}_1) = \left[\ \frac{dG^*(0,\ \beta_n)}{d\hat{r}_2}\ F^*\ (\hat{i}_1,\ \beta_n)\right] / \sqrt{|N^*_n|} \qquad (3.38)$$

$$E_{2,n}\ (\hat{r}_2) = \left[K\ \frac{dF\ (1,\ \quad_n)}{d\hat{r}_1}\ G\ (\hat{r}_2,\ \lambda_n)\right] / \sqrt{|N_n|} \qquad (3.39)$$

$$E^*_{2,n}\ (\hat{r}_2) = \left[K\ \frac{dF^*(1,\ \beta_n)}{d\hat{r}_1}\ G^*(\hat{r}_2,\ \beta_n)\right] / \sqrt{|N^*_n|} \qquad (3.40)$$

where N_n and N^*_n are defined by following equations for normalizing factors

[3 10] NORMALIZING FACTORS

$$N_n = [-\frac{dG(0, \lambda_n)}{d\hat{r}_2}]^2 B_1 \{[F(\hat{r}_1, \lambda_n)]^2\}$$

$$- HK^2 [\frac{dF(1, \lambda_n)}{d\hat{r}_1}]^2 B_2 \{[G(\hat{r}_2, \lambda_n)]^2\} \quad (3\ 41)$$

and

$$N_n^* = [\frac{dG^*(0, \beta_n)}{d\hat{r}_2}]^2 B_1 \{[F^*(\hat{r}_1, \beta_n)]^2\}$$

$$- HK^2 [\frac{dF^*(1, \beta_n)}{d\hat{r}_1}]^2 B_2 \{[G^*(\hat{r}_2, \beta_n)]^2\} \quad (3\ 42)$$

where n = 1, 2, 3, and $B_1 \{\ \}$ are defined by Eq (A 35) and (A 36) as bulk average temperatures over cross-section.

[3 11] EXPANSION COEFFICIENTS

The expansion coefficients A_n and C_n are defined by the following set of infinite linear algebraic equations

$$\sum_{k=1}^{\infty} [B_1\{E_{1,n}^* (\hat{r}_1) E_{1,k}^*(\hat{r}_1)\} (1-e^{-\beta_k^2 \hat{L}}) A_k$$

$$- B_1 \{E_{1,n}^* (\hat{r}_1) E_{1,k} (\hat{r}_1)\} (1-e^{-\lambda_k^2 \hat{L}}) C_k]$$

$$= B_1 \{E_{1,n}^* (\hat{r}_1)\} \quad (3\ 43)$$

$$\sum_{k=1}^{\infty} [-B_1 \{E_{1,n} (\hat{r}_1) E^*_{1,k} (\hat{r}_1)\} (1-e^{-\beta_k^2 \hat{L}}) A_k$$

$$+ B_1 \{E_{1,n} (\hat{r}_1) E_{1,k} (\hat{r}_1)\} (1-e^{-\lambda_k^2 \hat{L}}) C_k]$$

$$= - B_1 \{E_{1,n} (\hat{r}_1)\} \qquad (3\ 44)$$

with $n = 1,2,3,$

and for the zeroth order coefficient

$$C_o = \frac{1}{N_o} \sum_{k=1}^{\infty} [B_1 \{E^*_{1,n} (\hat{r}_1)\} (1-e^{-\beta_k^2 \hat{L}}) A_k$$

$$- B_1 \{E_{1,k} (\hat{r}_1)\} (1-e^{-\lambda_k^2 \hat{L}}) C_k] - \frac{H}{N_o} \qquad (3\ 45)$$

where $N_o = 1-H$

Numerical procedure to determine expansion coefficients is discussed in a later chapter.

[3.12] GENERAL HEAT BALANCE

A heat balance applied to the heat exchanger at any point along its length yields following relations for bulk temperatures.

$$\hat{T}_{1av} (\hat{z}) = H [\hat{T}_{2av} (\hat{z}) - \hat{T}_{2av} (0)] \qquad (3\ 46)$$

Hence at $\hat{z} = \hat{L}$

$$\hat{T}_{1av} (\hat{L}) = H [\hat{T}_{2av} (\hat{L}) - \hat{T}_{2av} (0)] \qquad (3\ 47)$$

Expressing bulk temperatures in series solutions, the above equations can be manipulated using orthogonality condition Eq (A 33) and Eq (A 34) to yield following result

$$\hat{T}_{1av}(\hat{z}) = (1-H)\, C_o + H\, \hat{T}_{2av}(\hat{z}) \tag{3 48}$$

At the ends of heat exchanger the above equation becomes,

$$\hat{T}_{1av}(\hat{L}) = (1-H)\, C_o + H \tag{3 49}$$

and $$\hat{T}_{2av}(0) = \frac{H-1}{H} C_o \tag{3 50}$$

[3 13] HEAT EXCHANGER EFFICIENCY

The efficiency of counter flow heat exchanger is defined as the ratio of actual heat transferred to that which would be transferred if the heat exchanger were infinitely long

$$\text{Actual heat transferred} = c_1 W_1 [T_{10} - T_{1av}(L)] \tag{3 51}$$

$$= c_2 W_2 [T_{2av(0)} - T_{2L}] \tag{3 52}$$

Heat transferred for infinite length

$$= c_1 W_1 [T_{10} - T_{2L}] \tag{3 53}$$

Ratio of preceding quantities yields

$$H < 1 \qquad \varepsilon = \frac{\hat{T}_{1av}(\hat{L})}{H} = 1 - \hat{T}_{2av(0)} \tag{3 54}$$

$$H > 1 \qquad \varepsilon = \hat{T}_{1av(\hat{L})} = H[1 - \hat{T}_{2av(0)}] \tag{3 55}$$

or using Eq (3 49) and (3 50)

$$H < 1 \qquad \varepsilon = \frac{1 - H}{H} C_o + 1 \tag{3 56}$$

$$H > 1 \qquad \varepsilon = (1-H)\, Co + H \tag{3 57}$$

physically as length of heat exchanger increases (i e approaches infinity) efficiency must approach to unity Hence as $\hat{L} \to \infty$, $\varepsilon \to 1$ Therefore from Eq (3 54) to (3 57)

$$H < 1 \qquad \text{as } \hat{L} \to \infty, \quad \hat{T}_{1av}(\hat{L}) \to H,$$

$$\hat{T}_{2av}(0) \to 0, \quad C_o \to 0 \tag{3 58}$$

$$H > 1 \qquad \text{as } \hat{L} \to \infty, \quad \hat{T}_{1av}(\hat{L}) \to 1,$$

$$\hat{T}_{2av}(0) \to H\text{-}1/H \; ; \quad C_o \to 1 \tag{3 59}$$

The series expansions for the end bulk temperatures may be written as,

$$\hat{T}_{1av}(\hat{L}) = C_o + \sum_{n=1}^{\infty} [A_n B_1 \{E^*_{1,n}(\hat{r}_1)\}$$

$$+ C_n B_1 \{E_{1,n}(\hat{r}_1)\} \; e^{-\lambda_n^2 \hat{L}}] \tag{3 60}$$

$$\hat{T}_{2av}(0) = C_o + \sum_{n=1}^{\infty} [A_n B_2 \{E^*_{2,n}(\hat{r}_2)\} e^{-\beta_n^2 \hat{L}}$$

$$+ C_n B_2 \{E_{2,n}(\hat{r}_2)\}] \tag{3 61}$$

Letting $\hat{L} \to \infty$ and applying Eq (3 58) to (3 61) and Eq (3 59) to Eq (3 60),

$$H < 1 \qquad \text{as } \hat{L} \to \infty, \quad \sum_{n=1}^{\infty} C_n \, B_2 \, \{E_{2,n} \, (\hat{r}_2)\} \quad \to \quad 0 \qquad (3\ 62)$$

$$H > 1 \qquad \text{as } \hat{L} \to \infty; \quad \sum_{n=1}^{\infty} A_n \, B_1 \, \{E^*_{1,n} \, (\hat{r}_1)\} \quad \to \quad 0 \qquad (3\ 63)$$

These equations reveal little about individual behaviour of the coefficients A_n and C_n as $\hat{L} \to \infty$ However, computations have revealed that the terms in each of the above summations are identical in sign for all values of n; therefore, since summations approach zero as $\hat{L} \to \infty$

$$H < 1 \qquad C_n \to 0 \qquad \text{as} \qquad \hat{L} \to \infty \qquad (3\ 64)$$

$$H > 1 \qquad A_n \to 0 \qquad \text{as} \qquad \hat{L} \to \infty \qquad (3\ 65)$$

[3 14] LONG HEAT EXCHANGER

For long heat exchangers i e., $L \to \infty$ Eq (3.64) and (3.65) reduces series expansions for the temperature distributions of the heat exchanger fluids

$$H < 1 \qquad \hat{T}_1(\hat{r}_1, \hat{z}) = C_o + \sum_{n=1}^{\infty} A_n \, E^*_{1,n} \, (\hat{r}_1) \, e^{-\beta_n^2(\hat{L}-\hat{z})} \qquad (3\ 66)$$

$$\hat{T}_2(\hat{r}_2, \hat{z}) = C_o + \sum_{n=1}^{\infty} A_n E^*_{2,n} (\hat{r}_2) e^{-\beta_n^2(\hat{L}-\hat{z})} \tag{3.67}$$

(3 67)

H > 1 $$\hat{T}_1(\hat{r}_1, \hat{z}) = C_o + \sum_{n=1}^{\infty} C_n E_{1,n} (\hat{r}_1) e^{-\lambda_n^2 \hat{z}} \tag{3 68}$$

$$\hat{T}_2(\hat{r}_2, \hat{z}) = C_o + \sum_{n=1}^{\infty} C_n E_{2,n} (\hat{r}_2) e^{-\lambda_n^2 \hat{z}} \tag{3 69}$$

For fully developed region only first term of the above expansions are required as other terms are negligible in magnitude compared to first term

[3 15] FULLY DEVELOPED REGION

For fully developed region temperature distribution is determined accurately enough retaining only first term in series expansion

H < 1 $$\hat{T}_1(\hat{r}_1, \hat{z}) \approx C_o + A_1 E^*_{1,1} (\hat{r}_1) e^{-\beta_1^2(\hat{L}-\hat{z})} \tag{3 70}$$

$$\hat{T}_2(\hat{r}_2, \hat{z}) \approx C_o + A_1 E^*_{2,1} (\hat{r}_2) e^{-\beta_1^2(\hat{L}-\hat{z})} \tag{3 71}$$

H > 1 $$\hat{T}_1(\hat{r}_1, \hat{z}) \approx C_o + C_1 E_{1,1} (\hat{r}_1) e^{-\lambda_1^2 \hat{z}} \tag{3.72}$$

$$\hat{T}_2(\hat{r}_2, \hat{z}) \approx C_o + C_1 E_{2,1} (\hat{r}_2) e^{-\lambda_1^2 \hat{z}} \tag{3 73}$$

Liquid metals have large thermal entry lengths compared to nonmetallic fluids Fully developed region for liquid metals may not be realized for small or moderately long heat exchangers Even for long heat exchangers, for accurate description of temperature at entry and exit, more number of terms are required

CHAPTER 4

PLUG FLOW IDEALIZATION

[4 1] INTRODUCTION

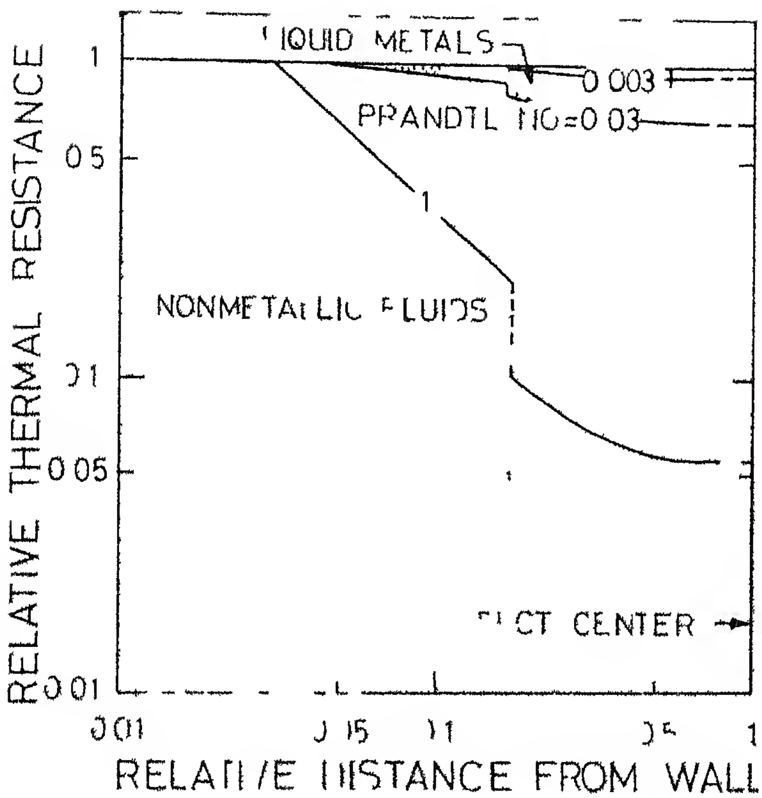
In principle assuming that sufficiently accurate values of $f_1(\hat{r}_1)$ and $g_1(\hat{r}_1)$ can be found from recent experimental results for liquid metals, the Eq. (3 34) to (3 37) can be integrated with stated boundary conditions Thus auxiliary functions F, F^*, G and G^* may be obtained This may not be straight forward as the differential equations may not submit to analytical solution. Expansion coefficients will have to be integrated numerically Obviously this would not desirable if it could be avoided. Much of engineering research can be associated with attempts to discover simplifications and convenient computational procedures

For turbulent flow of fluids, considerable simplifications are offered if plug flow model is assumed Then it does not become necessary to resort to numerical techniques for determining auxiliary functions, in our problem

[4.2] PLUG FLOW MODEL

The plug flow model involves the assumption of uniform velocity profile i e , $g_1(\hat{r}_1) = 1$ and if in addition we neglect effect of eddy diffusivity i e , $f_1(\hat{r}_1) = 1$, it is possible to

FIG 4 1



Relative t mal resistance
, olds

obtain analytical solution of the Eq (3 34) to Eq (3 37) With these approximations the equations for energy balance take simplified form for region 1 and 2 In fact with plug flow and $f_1(\hat{r}_1) = 1$, Eq (3 34) to Eq (3 37) take form of familiar Bessel equations and a relatively simple expression is obtained for F, F , G and G^*

Heat transfer analysis made with plug flow model for nonmetallic fluids give inaccurate results Stein (1966) has shown that with liquid metals this modelling yields meaningful and fairly accurate results, for low Peclet number ($Pe \leq 100$) Peclet number of 100 normally corresponds to a Reynolds number of 30,000 This range of Reynolds number for heat exchangers is definitely of practical interest Plug flow idealization is appropriate for liquid metals because steep temperature gradients are not localized near wall so that the error resulting from using uniform velocity profile i e , $g_1(\hat{r}_1) = 1$, in this region is not significant For low Peclet number ($Pe \leq 100$) the assumption of $f_1(\hat{r}_1) = 1$ is appropriate because eddy conduction plays a minor role compared to molecular conduction in the transfer of heat This is demonstrated by Fig (4 1) In our case $Pe > 100$ and hence an extension of this model will be needed This is discussed in Section [4 11]

[4 3] PLUG FLOW SOLUTIONS

With $g_1(\hat{r}_1) = f_1(\hat{r}_1) = 1$, Eq (3 34) to (3 37) become

$$\frac{d}{d\hat{r}_1}\left(\hat{r}_1 \frac{dF(\hat{r}_1, \lambda_n)}{d\hat{r}_1}\right) + \lambda_n^2 \hat{r}_1 F(\hat{r}_1, \lambda_n) = 0 \qquad (4\ 1)$$

$$\frac{dF(0, \lambda_n)}{d\hat{r}_1} = 0, \qquad F(0, \lambda_n) = 1$$

$$\frac{d}{d\hat{r}_1}\left(\hat{r}_1 \frac{dF^*(\hat{r}_1, \beta_n)}{d\hat{r}_1}\right) - \beta_n^2 \hat{r}_1 F^*(\hat{r}_1, \beta_n) = 0 \qquad (4\ 2)$$

$$\frac{dF^*(0, \beta_n)}{d\hat{r}_1} = 0, \qquad F^*(0, \beta_n) = 1$$

$$\frac{d}{d\hat{r}_2}\left((\hat{r}_2 + \sigma) \frac{dG(\hat{r}_2, \lambda_n)}{d\hat{r}_2}\right) - \lambda_n^2 \omega^2(\hat{r}_2 + \sigma) G(\hat{r}_2, \lambda_n) = 0$$

$$\frac{dG(1, \lambda_n)}{d\hat{r}_2} = 0; \qquad G(1, \lambda_n) = 1$$

$$\frac{d}{d\hat{r}_1}\left((\hat{r}_2 + \sigma) \frac{dG^*(\hat{r}_2, \beta_n)}{d\hat{r}_2}\right) + \beta_n^2 \omega^2(\hat{r}_2 + \sigma) G^*(\hat{r}_2, \beta_n) = 0$$

(4 4)

$$\frac{dG^*(1, \beta_n)}{d\hat{r}_2} = 0; \qquad G^*(1, \beta_n) = 1$$

Solutions of Eq (4 1) to (4 4) in terms of F, F^* , G and G^* are,

$$F(\hat{r}_1, \lambda_n) = J_o(\lambda_n \hat{r}_1) \qquad (4\ 5)$$

$$F^*(\hat{r}_1, \beta_n) = I_o(\beta_n \hat{r}_1) \qquad (4\ 6)$$

$$G(\hat{r}_2, \lambda_n) = \omega \lambda_n (1+\sigma)(K_1[\omega \lambda_n(1+\sigma)]I_o[\omega \lambda_n(\hat{r}_2+\sigma)] + I_1[\omega\lambda_n(1+\sigma)] K_o[\omega\lambda_n(\hat{r}_2+\sigma)]) \qquad (4\ 7)$$

$$G^*(\hat{r}_2, \beta_n) = \frac{\pi}{2} \omega \beta_n (1+\sigma) (J_1[\omega\beta_n(1+\sigma)]Y_o[\omega\beta_n(\hat{r}_2+\sigma)] + Y_1[\omega\beta_n(1+\sigma)] J_o[\omega\beta_n(\hat{r}_2+\sigma)]) \qquad (4\ 8)$$

where,

J_1, J_o, Y_1, Y_o	Bessel functions of first and second kind
I_1, I_o, K_1, K_o	Modified Bessel functions of first and second kind

Substitution of above expressions into Eq (3 30) and (3.31) yields eigenvalues Eigen functions can be obtained from Eq. (3 37) to (3 40) Finally expansion coefficients from (3 43) and (3 44)

[4 4] NARROW ANNULUS APPROXIMATION

A useful simplification is obtained letting $R \rightarrow 1$ This physically means a narrow annulus. As $R \rightarrow 1$, $\sigma \rightarrow \infty$ We find that with $R = 1$, F and F^* remain unaffected Eq (4.7) and (4.8) can be simplified letting $\sigma \rightarrow \infty$ and retaining only first term in large arguments approximations for Bessel functions A simpler approach is letting $\sigma \rightarrow \infty$ in Eq (4 3) and (4 4) which leads to following equations

$$\frac{d^2G(\hat{r}_2,\ \lambda_n)}{dr_2} - \omega^2 \lambda_n{}^2 G(\hat{r}_2,\ \lambda_n) = 0 \qquad (4.9)$$

$$\frac{d^2G^*(\hat{r}_2,\ \beta_n)}{d\hat{r}_2} + \omega^2 \beta_n{}^2 G^* (\hat{r}_2,\ \beta_n) = 0 \qquad (4\ 10)$$

with initial conditions remaining unchanged

It was found that by both techniques the expressions for functions G and G^* obtained were exactly same Numerical computations have revealed that for R as low as 0 5, error in computing efficiency is less than 3.5 percent For present case $R = 0\ 636$, therefore we adopt narrow annulus approximation approach The simplifications offered by employing narrow annulus approach can definitely be used to solve the problem, because the errors contained in the final results are insignificant

[4 5] AUXILIARY FUNCTIONS

For narrow annulus approximation the auxiliary functions obtained are as follows

$$F\ (\hat{r}_1,\ \lambda_n)\ =\ J_o\ (\lambda_n\ \hat{r}_1) \tag{4 11}$$

$$F^*(\hat{r}_1,\ \beta_n)\ =\ I_o\ (\beta_n,\ \hat{r}_1) \tag{4 12}$$

$$G\ (\hat{r}_2,\ \lambda_n)\ =\ \mathrm{Cosh}\ [\omega\lambda_n\ (1-\hat{r}_2)] \tag{4 13}$$

$$G^*(\hat{r}_2,\ \beta_n)\ =\ \mathrm{Cos}\ [\omega\beta_n(1-\hat{r}_2)] \tag{4 14}$$

For narrow annulus approximation the auxiliary functions are considerably simplified. Also R is eliminated except that it occurs in nondimensional parameter K

[4 6] EIGENVALUE EQUATIONS

Substituting F, F^*, G and G^* in eigenvalue Eq (3 30) and (3 31) we obtain,

$$\phi\ (\lambda_n)\ = \omega\ K_w\ \lambda_n{}^2 J_1(\lambda_n)\ \sinh\ (\omega\lambda_n) - \omega\ \lambda_n\ J_o(\lambda_n)$$
$$\sinh\ (\omega\lambda_n) + K\ \lambda_n J_1(\lambda_n)\ \cosh\ (\omega\lambda_n) \tag{4 14}$$

and

$$\phi^*(\beta_n)\ =\ \omega K_w\ \beta_n{}^2\ I_1\ (\beta_n)\ \sin\ (\omega\beta_n) + \omega\beta_n\ I_o(\beta_n)$$
$$\sin\ (\omega\beta_n) - K\beta_n\ I_1\ (\beta_n)\ \cos\ (\omega\beta_n) \tag{4.15}$$

[4 7] NORMALIZING FACTORS

From equations (3 41) and (3 42) normalizing factors are given as,

$$N_n = \omega^2 \lambda_n^2 \sinh^2(\omega\lambda_n)\,[J_o^2(\lambda_n) + J_1^2(\lambda_n)] - \tfrac{1}{2} HK^2 \lambda_n^2 J_1(\lambda_n)\,[1 + \frac{\sinh(2\omega\lambda_n)}{2\omega\lambda_n}] \tag{4 16}$$

$$N_n^* = \omega^2 \beta_n^2 \sin^2(\omega\beta_n)\,[I_o^2(\beta_n) - I_1^2(\beta_n)] - \tfrac{1}{2} HK^2 \beta_n^2 I_1(\beta_n)\,[1 + \frac{\sin(2\omega\beta_n)}{2\omega\beta_n}] \tag{4 17}$$

[4 8] NORMALIZED EIGENFUNCTIONS

$$E_{1,n}(\hat{r}_1) = -\omega\lambda_n \sinh(\omega\lambda_n)\, J_o(\lambda_n \hat{r}_1)/\sqrt{|N_n|} \tag{4.18}$$

$$E^*_{1,n}(\hat{r}_1) = \omega\beta_n \sin(\omega\beta_n)\, I_o(\beta_n \hat{r}_1)/\sqrt{|N_n^*|} \tag{4.19}$$

$$E_{2,n}(\hat{r}_2) = -K\lambda_n J_1(\lambda_n) \cosh[\omega\lambda_n(1-\hat{r}_2)]/\sqrt{|N_n|} \tag{4.20}$$

$$E^*_{2,n}(\hat{r}_2) = K\beta_n I_1(\beta_n) \cos[\omega\beta_n(1-\hat{r}_2)]/\sqrt{|N_n^*|} \tag{4 21}$$

[4.9] EXPANSION COEFFICIENTS

In Eq (3 43) and (3 44) various factors for plug flow are given as follows

$$B_1\{E^*_{1,n}(\hat{r}_1)\, E^*_{1,k}(\hat{r}_1)\} = 2\int_0^1 E^*_{1,n}(\hat{r}_1)\, E^*_{1,k}(\hat{r}_1)\, \hat{r}_1\, d\hat{r}_1$$

$$= \frac{\omega^2 \beta_n^{\ 2} \sin^2(\omega\beta_n)\,[I_o^{\ 2}(\beta_n) - I_1^{\ 2}(\beta_n)]}{|N_n^*|} + \frac{-e^{+\beta_n^{\ 2}\hat{L}}}{1-e^{+\beta_n^{\ 2}\hat{L}}}$$

$$\text{for } n = k \qquad (4\ 22)$$

$$= 2\ [\omega\beta_n \sin(\omega\beta_n)][\ \beta_k \sin(\omega\beta_k)][\beta_k I_1(\beta_k) I_o(\beta_n) - \beta_n I_1(\beta_n)\, I_o(\beta_k)]/[[\beta_k^{\ 2} - \beta_n^{\ 2}]\sqrt{[|N_n^*|\ \ |N_k^*|\ ]}]$$

$$\text{for } n \neq k \qquad (4\ 23)$$

$$B_1\{L^*_{1,n}(\hat{r}_1)\, E_{1,k}(\hat{r}_1)\} = 2\int_0^1 L^*_{1,n}(\hat{r}_1)\, E_{1,k}(\hat{r}_1)\, \hat{r}_1\, d\hat{r}_1$$

$$= -\,2\,[\omega\beta_n \sin(\omega\beta_n)]\,[\ \omega\lambda_k \sinh(\omega\lambda_k)][\beta_n I_1(\beta_n) J_o(\lambda_k) + \lambda_k I_o(\beta_n)\, J_1(\lambda_k)]/((\sqrt{|N_n^*|\ \ |N_k|}\ )\,(\beta_n^{\ 2} + \lambda_k^{\ 2})) \qquad (4\ 24)$$

$$B_1\{E^*_{1,k}(\hat{r}_1)\, E_{1,n}(\hat{r}_1)\} = 2\int_0^1 E_{1,n}(\hat{r}_1)\, E^*_{1,k}(\hat{r}_1)\hat{r}_1\, d\hat{r}_1$$

$$= -\,2\,[\omega\beta_k \sin(\omega\beta_k)]\,[\omega\lambda_n \sinh(\omega\lambda_n)]\,[\beta_k I_1(\beta_k) J_o(\lambda_n) + \lambda_n I_o(\beta_k)\, J_1(\lambda_n)]/[\sqrt{(|N_n|\ \ |N_k^*|\ )}(\beta_k^{\ 2} + \lambda_n^{\ 2})] \qquad (4.25)$$

$$B_1 \{E_{1,n}(\hat{r}_1)\, E_{1,k}(\hat{r}_1) = 2 \int_0^1 E_{1,n}(\hat{r}_1)\, E_{1,k}(\hat{r}_1)\hat{r}_1\, d\hat{r}$$

$$= [\omega\lambda_n \sinh(\omega\lambda_n)]^2 [J_o^{\,2}(\lambda_n) + J_1^{\,2}(\lambda_n)] / |N_n|$$

$$+ e^{-\lambda_n^2 \hat{L}} / (1 - e^{-\lambda_n^2 \hat{L}}) \qquad \text{for } n = k \qquad (4\ 26)$$

$$= 2[\omega\lambda_n \sinh(\omega\lambda_n)]\, [\omega\lambda_k \sinh(\omega\lambda_k)]\, [\lambda_n J_1(\lambda_n)\, J_o(\lambda_k)$$

$$- \lambda_k J_1(\lambda_k)\, J_o(\lambda_n)] / [(\sqrt{|N_n|\ \ |N_k|}\ )(\lambda_k^{\,2} - \lambda_n^{\,2})]$$

$$\text{for } n \neq k \qquad (4\ 27)$$

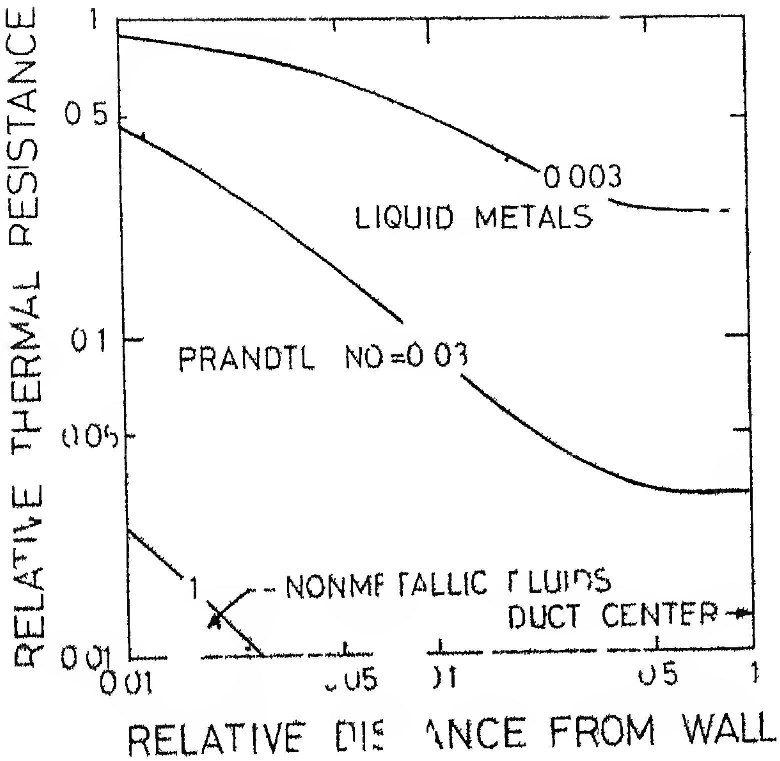
$$B_1 \{E^*_{1,n}(\hat{r}_1)\} = 2 \int_0^1 E^*_{1,n}(\hat{r}_1)\, \hat{r}_1\, d\hat{r}_1$$

$$= 2\, \frac{\omega\beta_n \sin(\omega\beta_n)\, I_1(\beta_n)}{(\sqrt{|N^*_n|}\ )\ \beta_n} \qquad (4\ 28)$$

$$B_1 \{E_{1,n}(\hat{r}_1)\} = 2 \int_0^1 E_{1,n}(\hat{r}_1)\, \hat{r}_1\, d\hat{r}_1$$

$$= -\,2\, \frac{\omega\lambda_n \sinh(\omega\lambda_n)\, J_1(\lambda_n)}{(\sqrt{|N_n|}\ )\ \lambda_n} \qquad (4\ 29)$$

FIG 4 2



mal resistance distribution

f 1

[4 10] BULK TEMPERATURE

Expressions for bulk temperatures are given as,

$$\hat{T}_{1av}(\hat{z}) = C_o + \sum_{n=1}^{\infty} [2 A_n \frac{\omega \sin(\omega\beta_n) I_1(\beta_n)}{\sqrt{|N_n^*|}} e^{-\beta_n^2(\hat{L}-\hat{z})}$$

$$- 2 C_n \frac{\omega \sinh(\omega\lambda_n) J_1(\lambda_n)}{\sqrt{|N_n|}} e^{-\lambda_n^2 \hat{z}}] \quad (4\ 30)$$

$$\hat{T}_{2av}(\hat{z}) = C_o + \sum_{n=1}^{\infty} [A_n \frac{K I_1(\beta_n) \sin(\omega\beta_n)}{\omega \sqrt{|N_n^*|}} e^{-\beta_n^2(\hat{L}-\hat{z})}$$

$$C_n \frac{K \sinh(\omega\lambda_n) J_1(\lambda_n)}{\omega \sqrt{|N_n|}} e^{-\lambda_n^2 \hat{z}}] \quad (4\ 31)$$

It can be verified that the orthogonality condition is satisfied for plug flow, narrow annulus (R = 1) case General heat balance is also satisfied

[4 11] PLUG FLOW EXTENSION FOR LARGE PECLET NUMBER RANGE

Liquid motal heat exchanger applications to nuclear tecnology involves very high flow rates Consecuently Reynolds numbers are large (Re No $\geq 10^5$) Peclet number corresponding to such range is much greater than 100.

For this range of Peclet number, in plug flow idealization, assuming $f_1(\hat{r}_1) = 1$ is inaccurate For Reynolds number of 10^6 there is a large variation in eddy diffusivity across

duct cross-section as is seen in Fig (4 2) Stein (1966) suggested an approximation to extend the validity of plug flow for peclet number range upto 1000 This approximation involves the use of a properly averaged space independent total conductivity which results when a simple variable change is made

The following derivations follows the method suggested by Stein (1966)

[4 12] DERIVATION

A variable change in radial vaiiable $\hat{r}_1$ is introduced The new variable $\bar{x}_1(\hat{r}_1)$ is related to $\hat{r}_1$ by a first order differential equation

Tube

$$\frac{d\bar{x}_1}{d\hat{r}_1} = \frac{\bar{x}_1}{\hat{r}_1} \frac{k_1^+}{f_1(\hat{r}_1)} \qquad (4\ 32)$$

Annulus

$$\frac{d\bar{x}_2}{d\hat{r}_2} = \left(\frac{\bar{x}_2 + \sigma}{\hat{r}_2 + \sigma} \right) \frac{k_2^+}{f_2(\hat{r}_2)} \qquad (4\ 33)$$

with the boundary condition $\bar{x}_1(0) = 0$ and with k_1^+ a constant, so chosen that $\bar{x}_1(1) = 1$

Introducing variable change in the equations (3 12) and (3 13); in terms of $\bar{x}_1$, they can now be written as follows

Tube

$$\frac{1}{\bar{x}_1}\frac{\partial}{\partial \bar{x}_1}\left[\tilde{x}_1 \frac{\partial \bar{T}_1(\bar{x}_1, \bar{z})}{\partial \bar{x}_1}\right] = g_1(\bar{x}_1) \frac{\partial \bar{T}_1(\bar{x}_1, \bar{z})}{\partial \bar{z}} \qquad (4\ 34)$$

Annulus

$$\frac{1}{(\bar{x}_2 + \sigma)}\frac{\partial}{\partial \bar{x}_2}\left[(\bar{x}_2 + \sigma) \frac{\partial \bar{T}_2(\bar{x}_2, \bar{z})}{\partial \bar{x}_2}\right]$$

$$- \omega^2 \frac{k_1^+}{k_2^+} g_2(\bar{x}_2) \frac{\partial \bar{T}_2(\bar{x}_2, \bar{z})}{\partial \bar{x}_2} \qquad (4.35)$$

where,

$$\bar{z} = k_1^+ \hat{z} \quad ; \text{ and } \quad \bar{L} = k_1^+ \hat{L} \qquad (4\ 36)$$

$$g_1(\bar{x}_1) = g_1(\hat{r}_1) \left(\frac{\hat{r}_1}{\bar{x}_1}\right)^2 \frac{d\hat{r}_1}{d\bar{x}_1} \qquad (4\ 37)$$

$$g_2(\bar{x}_2) = g_2(\hat{r}_2) \left(\frac{\hat{r}_2 + \sigma}{\bar{x}_1 + \sigma}\right)^2 \frac{d\hat{r}_2}{d\bar{x}_2} \qquad (4\ 38)$$

The boundary conditions become,

Entrance.

$$\bar{T}_1(\bar{x}_1, 0) = 0 \qquad (4.39)$$

$$\tilde{T}_2\ (\tilde{x}_2,\ \tilde{L})\ =\ 1 \qquad (4\ 40)$$

<u>Interior</u>

$$\frac{\partial \tilde{T}_1\ (0,\ \tilde{z})}{\partial \tilde{x}_1} = 0 \qquad (4\ 41)$$

$$K\ \frac{k_1^+}{k_2^+}\ \frac{\partial \tilde{T}_1\ (1,\ \tilde{z})}{\tilde{x}_1} = \frac{\partial \tilde{T}_2\ (0,\ \tilde{z})}{\partial \tilde{x}_2} \qquad (4\ 42)$$

$$K_w\ k_1^+\ \frac{\partial \tilde{T}_1(1,\tilde{z})}{\partial \tilde{x}_1} = \tilde{T}_2\ (0,\tilde{z}) - \tilde{T}_1(1,\tilde{z}) \qquad (4\ 43)$$

$$\frac{\partial \tilde{T}_2\ (1,\tilde{z})}{\partial \tilde{x}_2} = 0 \qquad (4\ 44)$$

For turbulent flow and all ranges of Pe No the $g(\tilde{x}_1)$ and $g(\tilde{x}_2)$ as defined by Eq (4 37) and (4 38) have average value over cross-section equal to unity For small Pe No $g(\tilde{x}_1)$ and $g(\tilde{x}_2)$ are approximately equal to unity for a large portion of cross-section of the heat exchanger duct

Liquid metals are less sensitive to the exact representation of velocity profile compared to eddy diffusivity representation, Stein (1966) Therefore as an approximation it will be assumed that $g(\tilde{x}_1)$ and $g(\tilde{x}_2)$ are identically equal to unity for all values of Peclet No. This reduces Eq (4 34) and (4 35) to following

Tube

$$\frac{1}{\bar{x}_1}\frac{\partial}{\partial \bar{x}_1}\left[\tilde{x}_1 \frac{\partial \bar{T}_1(\bar{x}_1, \bar{z})}{\partial \bar{x}_1}\right] = \frac{\partial \bar{T}_1(\bar{x}_1, \bar{z})}{\partial \bar{z}} \qquad (4\ 45)$$

Annulus

$$\frac{1}{\bar{x}_2 + \sigma}\frac{\partial}{\partial \bar{x}_2}\left[(\tilde{x}_2 + \sigma)\frac{\partial \bar{T}_2(\bar{x}_2, \bar{z})}{\partial \bar{x}_2}\right] = -\omega^2 \frac{\partial \bar{T}_2(\bar{x}_2, \bar{z})}{\partial \bar{x}_2} \qquad (4\ 46)$$

with boundary conditions remaining unchanged

Eq (4 45) and (4.46) are identical to Eq (3 12) and (3 13) when plug flow model i e , $f_1(\hat{r}_1) = g_1(\hat{r}_1) = 1$ is applied to later set of equations, and making following changes in non-dimensional parameters

$$K \rightarrow \frac{k_1^+}{k_2^+} K \qquad (4\ 47)$$

$$K_w \rightarrow k_1^+ K_w \qquad (4\ 48)$$

$$\bar{z} \rightarrow k_1^+ \hat{z} \qquad (4\ 49)$$

In effect, the k_1^+ approximation has changed molecular conductivity k_1 to k_1^+, in plug flow equations Hence the results obtained for plug flow without k_1^+ are valid for large Pe No approximation with above modification in parameters only

[4.13] Determination of k_1^+

Expressions for k_1^+ may be determined from integrating eqns (4.32) and (4.33) between 0 to 1

$$k_1^+ = \frac{\int_0^1 \frac{d\bar{x}_1(\hat{r}_1)}{\bar{x}_1(\hat{r}_1)}}{\int_0^1 \frac{1}{f_1(\hat{r}_1)} \frac{d\hat{r}_1}{\hat{r}_1}} \tag{4.50}$$

$$k_2^+ = \frac{\int_0^1 \frac{d\bar{x}_2(\hat{r}_2)}{(\bar{x}_2(\hat{r}_2)+\sigma)}}{\int_0^1 \frac{1}{f_2(\hat{r}_2)} \frac{d\hat{r}_2}{(\hat{r}_2+\sigma)}} \tag{4.51}$$

There are two complications in determining k_1^+. First it is necessary to know $f_1(\hat{r}_1)$ which essentially is knowledge of ε_{Hi} as a function of radial location. This can be obtained from experimental results. Second complication is that right hand sides of equations (4.50) and (4.51) take indeterminate form at $\hat{r}_i = 0$, i.e., $\bar{x}_1 = 0$

An alternative method is suggested by Stein (1966), according to which liquid metal heat transfer coefficients can be computed from plug flow values by simple relation

$$N_u(z) = k^+ [N_u(z)]_{PF} \tag{4.52}$$

where,

PF - Plug Flow

Nu - Nusselt Number

Since detailed temperature distributions are not required there is no need to determine $x_1(\hat{r}_1)$ from equations (4 32) and (4.33) Appropriate values of k_1^+ can be determined from equation (4 52) when applied to fully developed nusselt numbers either from experiment or by computation

The equations used for evaluating k_1^+ are ,

<u>Tube</u>

$$k_1^+ = (7 + 0\ 025\ Pe_1^{0\ 8})/8\ 0 \qquad (4\ 53)$$

<u>Annulus</u>

$$k_2^+ = (5\ 25 + 0\ 0188\ Pe_2^{0\ 8}\ (\frac{r_{22}}{r_{21}})^{0\ 3})/6\ 0 \qquad (4\ 54)$$

Eq (4 53) is obtained from correlations given by Lyon (1955) and Eq (4.54) from correlations given by Bailey (1950)

CHAPTER 5

RESULTS AND DISCUSSION

[5 1] EIGENVALUES AND EIGENFUNCTIONS

The square roots of the eigenvalues, λ_n^2 and β_n^2, were determined from the roots of the Eq (4 14) and (4 15) These nonlinear algebraic equations involve nondimensional parameters K, K_w and ω The values of these and other parameters for the Kalpakkam heat exchanger are given below To calculate these values thermal conductivities of sodium and austenitic stainless-steel were obtained from Raznjewi (1976),

$$Pe_1 = 160$$

$$Pe_2 = 190$$

$$k_1^+ = 1\ 055$$

$$k_2^+ = 1.11$$

$$K = (\ 5722)\ (k_1^+/k_2^+)$$

$$H = 1.65$$

$$K_w = (\ 5698)\ (k_1^+)$$

$$\tilde{z} = (2\ 6)(k_1^+)$$

The values of k_1^+ and k_2^+ were obtained from Eq (4 53), and (4.54). Numerical values given within the brackets are plug flow values

The Eq (4 14) and (4 15) having K, K_w and ω as multiplying constants were solved by Regula-False method The accuracy in determining the roots of these equations is of the order of 10^{-6} Once the eigenvalues are known it is easy to calculate eigenfunctions from Eq (4 18) to (4 21)

[5 2] EXPANSION COEFFICIENTS

The expansion coefficients were calculated from the Eq (4 22) to (4 27) and the nonhomogeneous part from Eq (4 28) and (4 29) To obtain A_n and C_n an infinite set of equations must be solved It has been shown by Brown (1968) that it is enough to consider only ten equations to obtain C_o accurate up to three decimal places

For long heat exchangers i e , $z \geq 1$, only first term was required to calculate temperature distribution The subsequent terms had negligible contribution except close to the entrance or exit of the heat exchanger

[5 3] COMPARISON OF PRESENT METHOD WITH NTU-METHOD

The performance of a conventional heat exchanger is usually predicted by the effectiveness - NTU method In this method, the calculation of NTU (Number of heat transfer units) involves the estimation of heat transfer coefficient for primary and secondary side, using experimental correlations

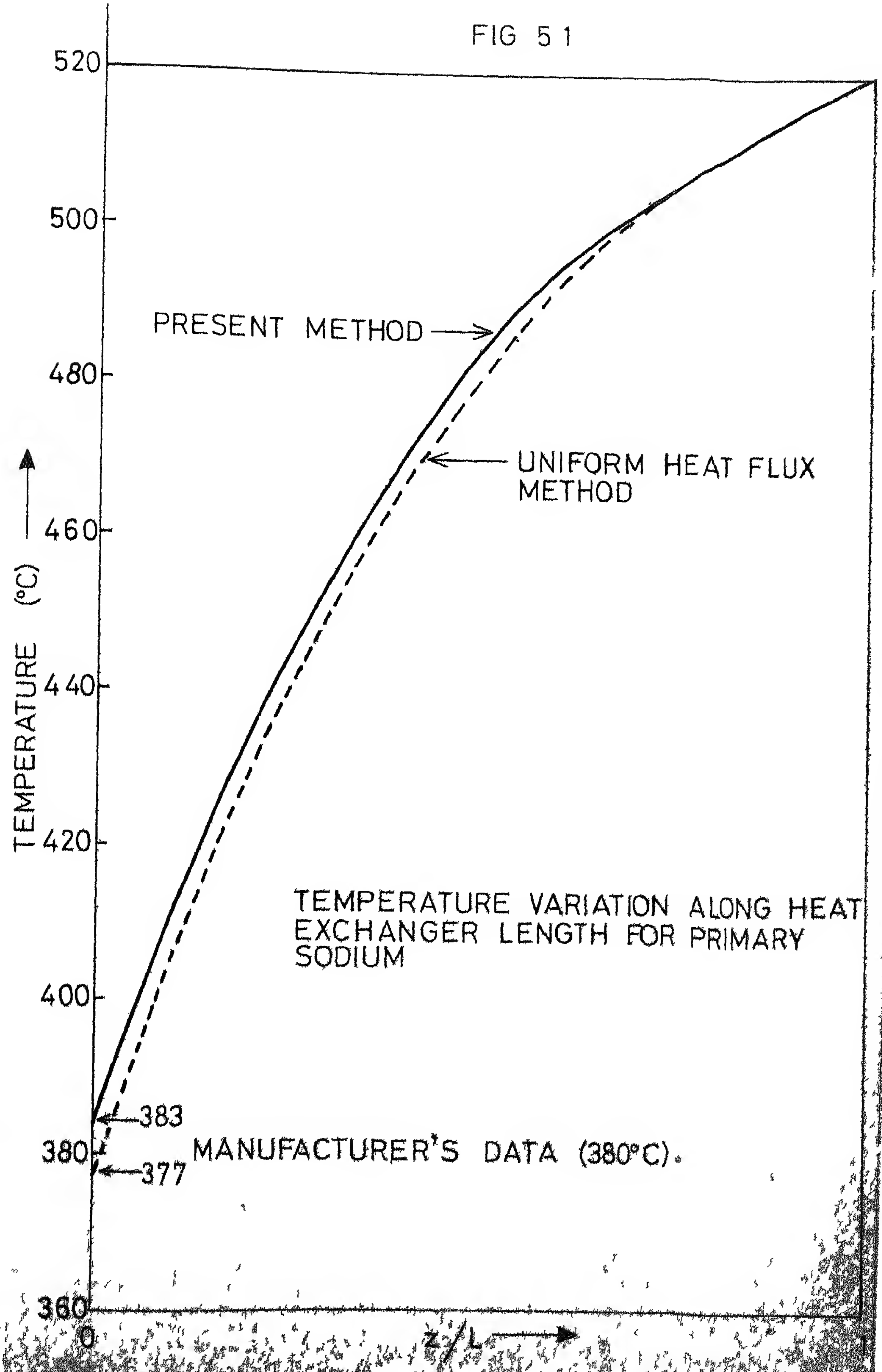
As already pointed out in Chapter 1, the heat transfer coefficient is a strong function of boundary conditions There are two extreme boundary conditions normally considered in heat transfer problems, viz , constant heat flux and constant temperature

In the case of flow inside tubes experimental correlations are available for both conditions but for flow in annulus, correlation for constant heat flux is only available. Hence we compare our results with the results obtained by NTU-Method, for constant heat flux boundary condition Table 5 1 compares the efficiency and exit temperature values for primary and secondary sodium as obtained by present method and uniform heat flux method For uniform heat flux (UHF) calculations see Appendix B.

Table 5 1

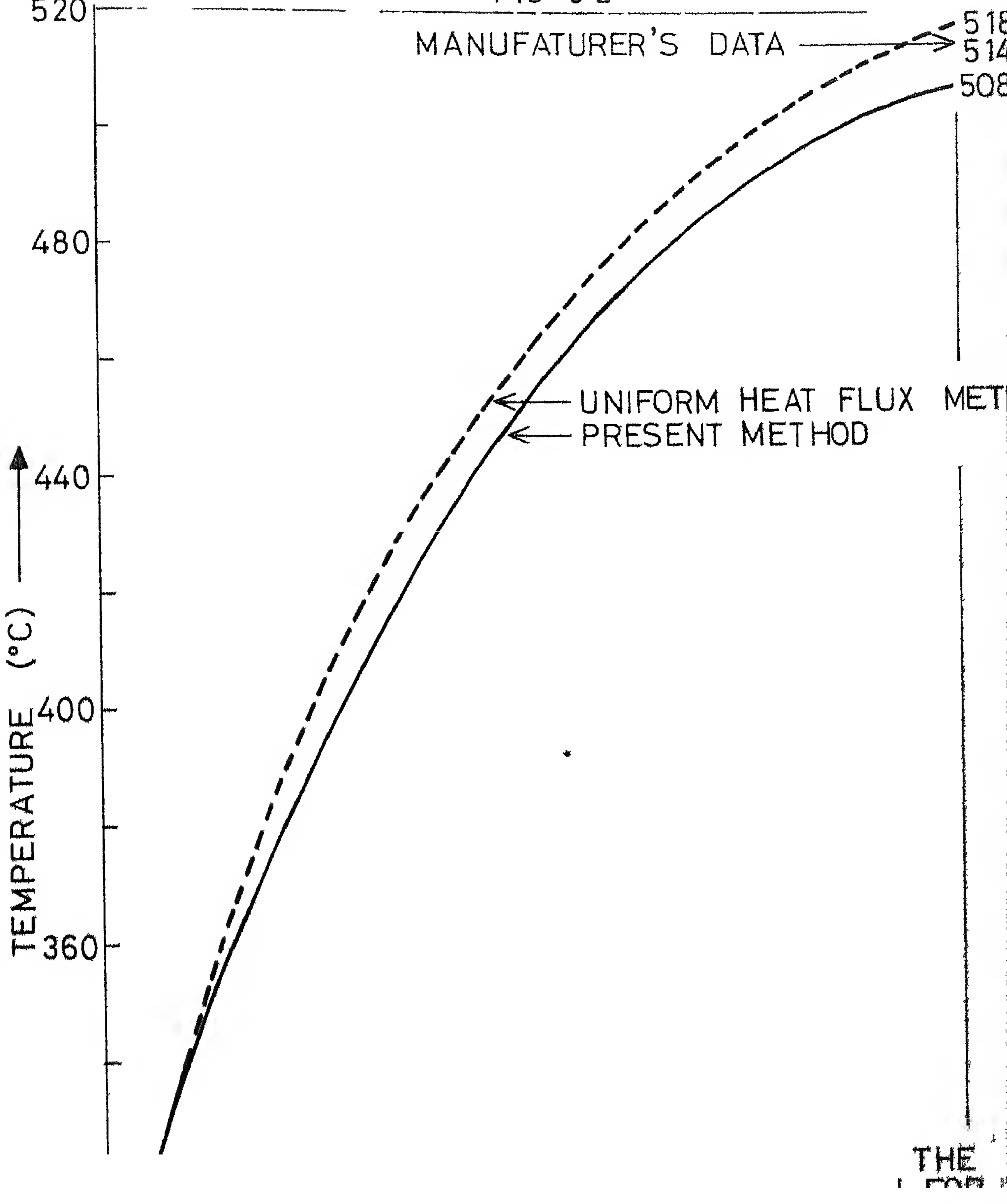
	Present Method	Manufacturer's Data	UHF Method
Efficiency	95 %	97 5 %	99 3 %
Exit Temperature for Primary Sodium	383°C	380°C	377°C
Exit Temperature for Secondary Sodium	508°C	514°C	518°C

FIG 5 1



520
500
480
460
440
420
400
380
360
TEMPERATURE (°C)
PRESENT METHOD
UNIFORM HEAT FLUX METHOD
TEMPERATURE VARIATION ALONG HEAT EXCHANGER LENGTH FOR PRIMARY SODIUM
383
377
MANUFACTURER'S DATA (380°C).
0
Z/L
1

FIG 52



MANUFATURER'S DATA
518
514
508
UNIFORM HEAT FLUX METH
PRESENT METHOD
520
480
440
400
360
TEMPERATURE (°C)
THE

The UHF approach over predicts efficiency, but the error is not serious because the actual efficiency quoted by manufacturer is close to hundred percent It is also not very clear that manufacturer's data represents experimental values or calculated values Table 5 1 also shows that UHF over predicts exit temperature for secondary sodium and under predicts exit temperature for primary sodium The present method shows the opposite trend Temperature variation for UHF and present method along length is shown in Fig (5 1) and Fig (5 2), The maximum temperature difference in two methods are at the outlet since the inlet temperatures were assumed to be same in both cases

Manufacturer's quoted temperature values are only available at present Actual temperature values will be known only after experiments conducted at Kalpakkam These experiments might throw light upon the actual obtainable exit temperature values At present the Intermediate heat exchanger is under fabrication by BHEL

[5 4] EFFECT OF VARIATION OF FLOW RATE

Under normal operating conditions for IHX, H = 1 65. A change in operating reactor power level will have a corresponding change in the quantity of heat transferred at IHX from primary to secondary sodium. If ~~reactor~~ reactor power level goes up more heat will be exchanged at IHX Consequently,

a higher secondary mass flow rate will be required to transport this heat If reactor power level goes down less heat is exchanged and a lower secondary mass flow rate will be required In either case H will vary as it represents ratio of primary sodium to secondary sodium heat capacity flow rates

It is reasonable to assume that primary flow rate and its exit temperature from the core is constant When power level goes up, the higher power is transferred to primary sodium inside core This can be achieved keeping same mass flow rate and exit temperature, from reactor core, with a lower inlet temperature to the core When power level reduces, other conditions remaining constant, a higher inlet temperature to the core is anticipated

Secondary sodium mass flow rate will depend upon the quantity of heat to be transferred But it is reasonable to consider exit temperature of secondary sodium from steam generator to be constant This is possible because a changed secondary mass flow rate will result in a change in the quantity of steam generated for same exit temperature from steam generator Only IHX exit temperature will increase or decrease depending upon increase or decrease in reactor power level This may be of practical interest also if in secondary loop due to blockage or additional resistances,

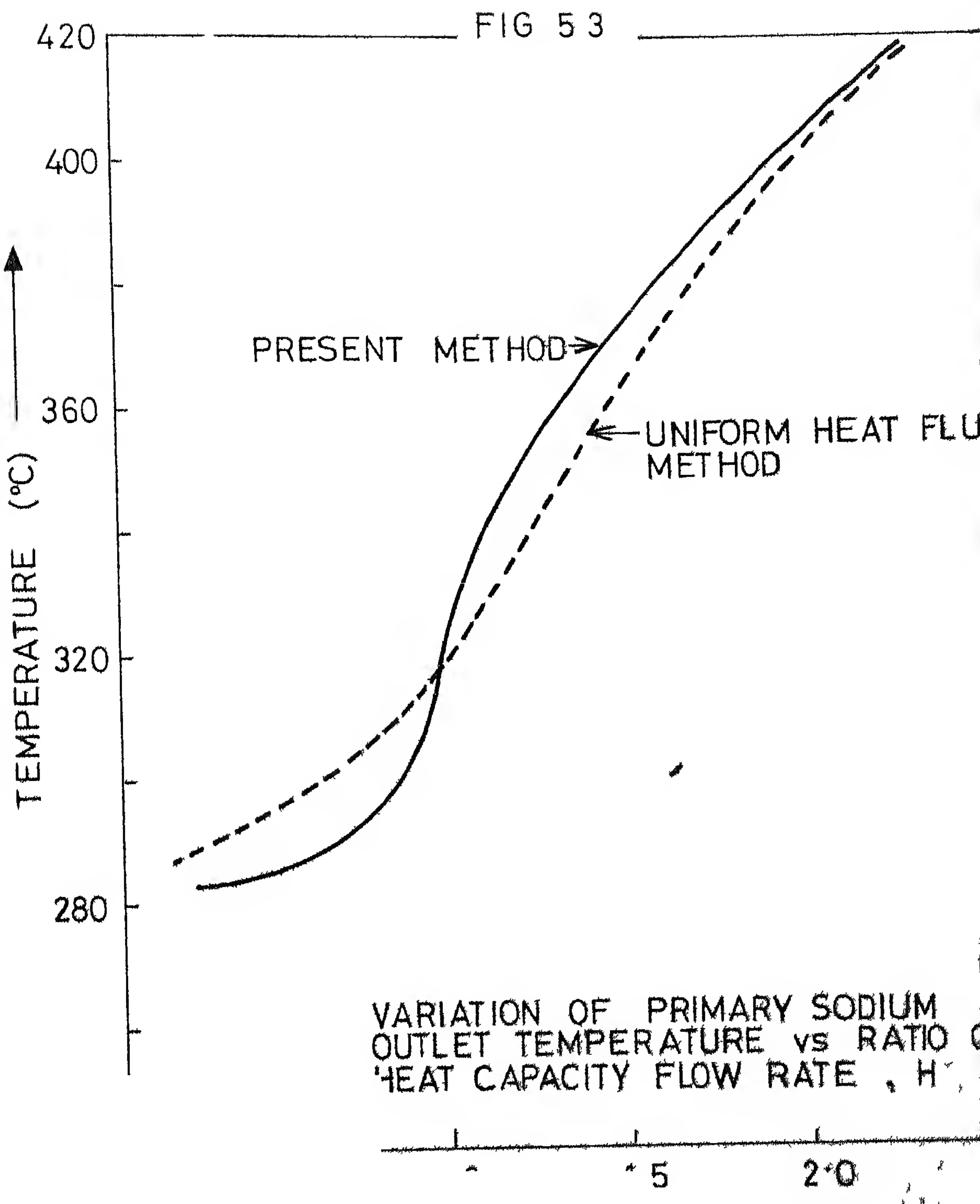
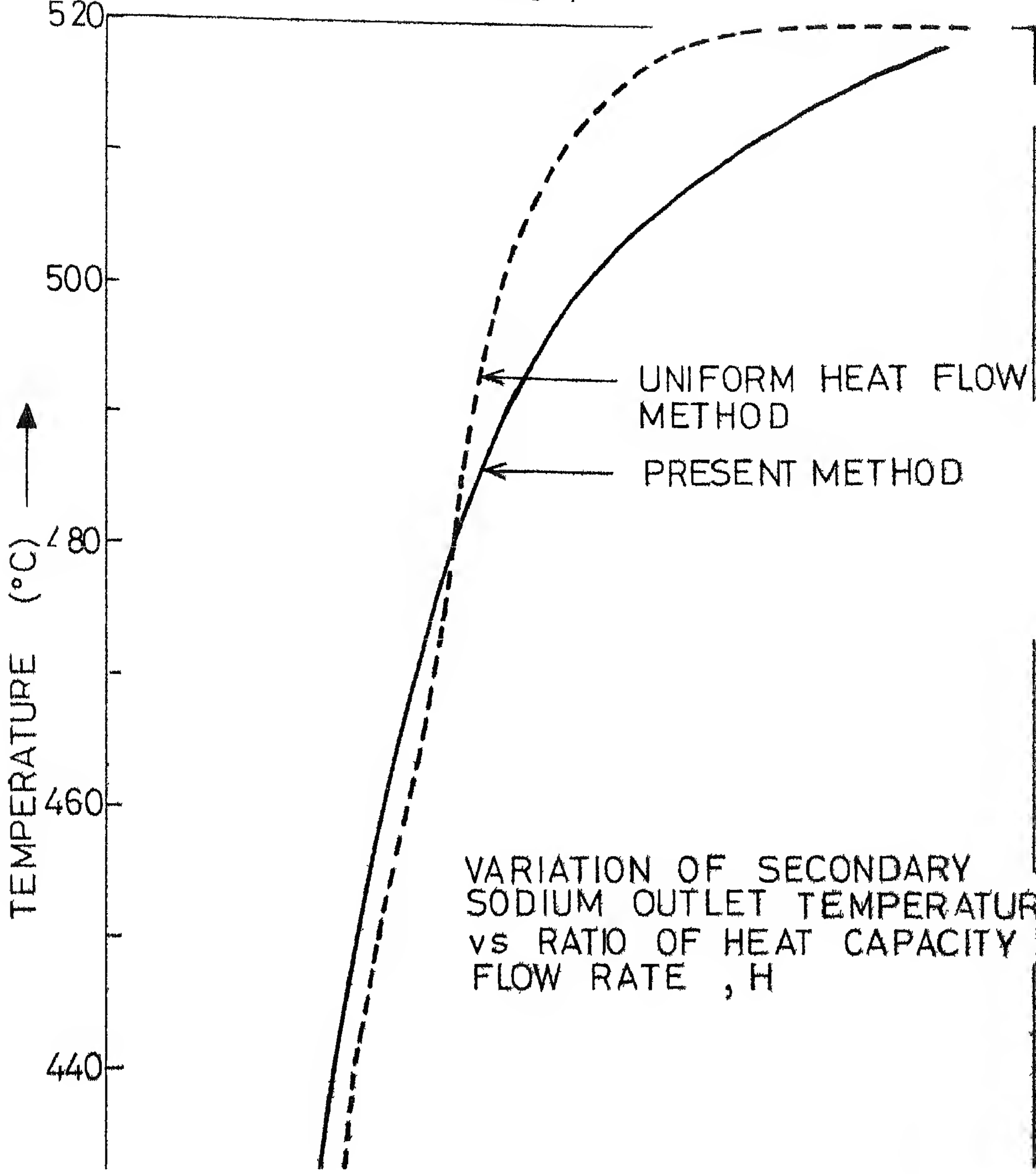


FIG 53
420
400
360
320
280
TEMPERATURE (°C)
PRESENT METHOD
UNIFORM HEAT FLU
METHOD
VARIATION OF PRIMARY SODIUM
OUTLET TEMPERATURE vs RATIO
HEAT CAPACITY FLOW RATE , H
5
2·0

FIG 5 4


520
500
480
460
440
TEMPERATURE (°C)
UNIFORM HEAT FLOW
METHOD
PRESENT METHOD
VARIATION OF SECONDARY
SODIUM OUTLET TEMPERATUR
vs RATIO OF HEAT CAPACITY
FLOW RATE , H

the secondary sodium mass flow rate changes Any change in secondary sodium flow rate due to either reason, will result in a corresponding change in H With the reasonable assumptions of keeping inlet temperatures to the IHX constant, exit temperatures for both primary and secondary sodium can be predicted, under steady state conditions Fig (5 3) and Fig (5 4) gives such a plot, both for present method and UHF method

As anticipated for higher values of H both predict higher exit temperatures for secondary sodium For H decreasing both methods predict decrease in secondary sodium exit temperature This trend is observed for primary sodium also

It is interesting to see that for $H > 1$ UHF over predic the exit temperature values for secondary sodium and under predicts for primary sodium, which essentially means over prediction of efficiency compared to the present method For $H < 1$, it under predicts efficiency A good agreement is obtained for $H = 1$ This means that for $H = 1$, UHF method and present method yield same results

For $H > 1$ and increasing H will decrease C_{min}, which for $H > 1$ is given by secondary heat capacity flow rate Since NTU is defined as $\bar{U}A/C_{min}$, decreasing C_{min} will increase NTU (See Appendix B) and from Eq (B 6) it is obviou

that efficiency will increase This increase in efficiency is much higher for UHF compared to present method for equal increase in H. Hence UHF method over predicts temperatures

For $H < 1$ and decreasing H will fix C_{min} because now C_{min} corresponds to primary heat capacity flow rate which is considered constant From Eq (B 5), it is clear that denominator does not decrease as fast as numerator Consequently efficiency increase for $H < 1$ and decreasing H is at a slower rate than for $H > 1$ and increasing H This resulted in under-prediction of efficiency for $H < 1$ by UHF method compared to present method

For $H = 1$, for a heat exchanger energy balance equation at any point along the length predicts constant bulk temperature difference between primary and secondary sodium This means that there is a constant heat flux all along the length Therefore UHF should predict accurate results under such condition Present method also yields same results and therefore it is advisable to use NTU method when $H = 1$, based on UHF conditions

[5 5] COMPARISON OF k_1^+ WITH PLUG FLOW MODEL

The Fig. (5 1) to Fig (5.4) were presented using k_1^+ approximation. Neglecting k_1^+ in the present case results in the under prediction of primary sodium exit temperature by

FIG 55

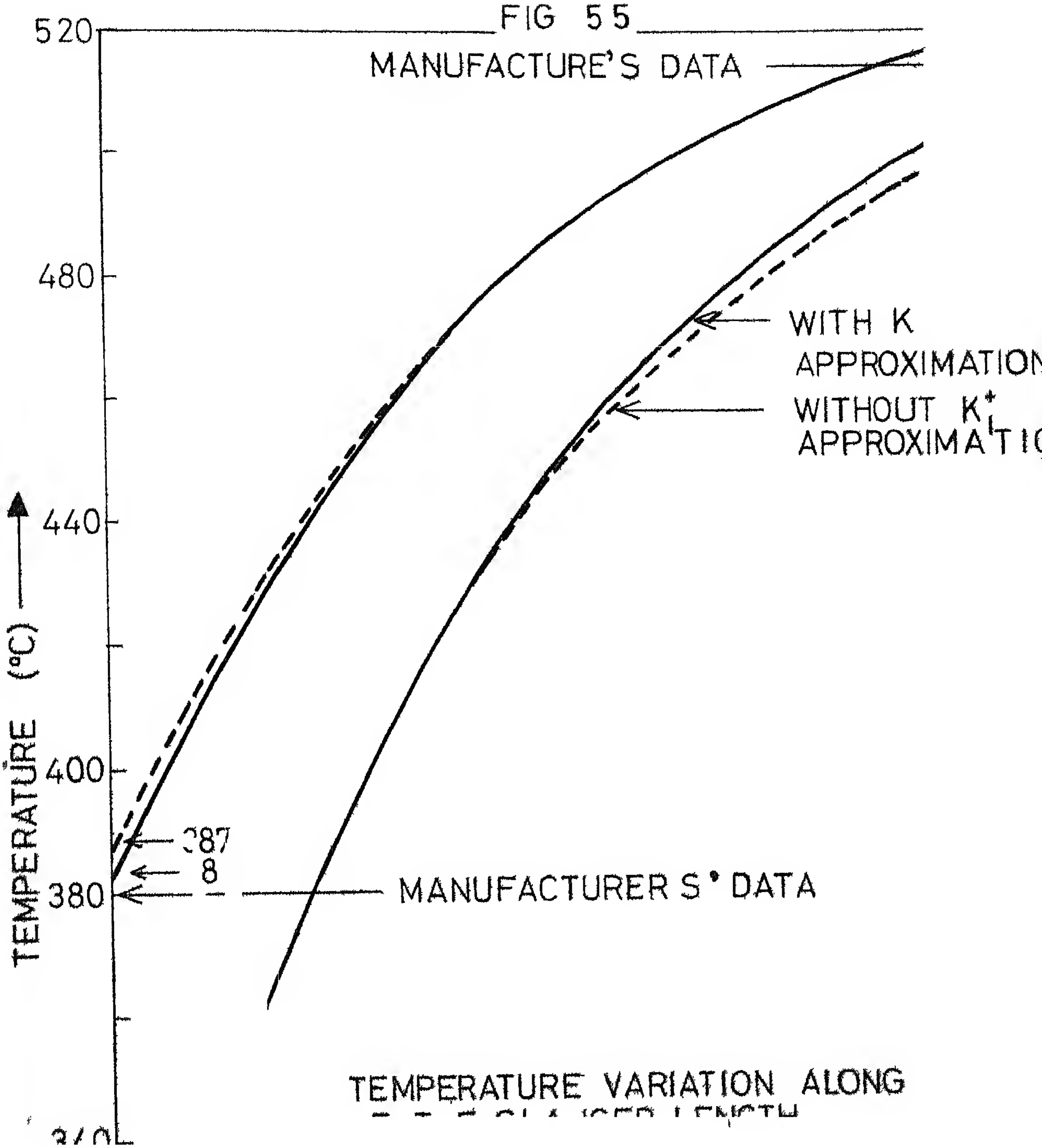
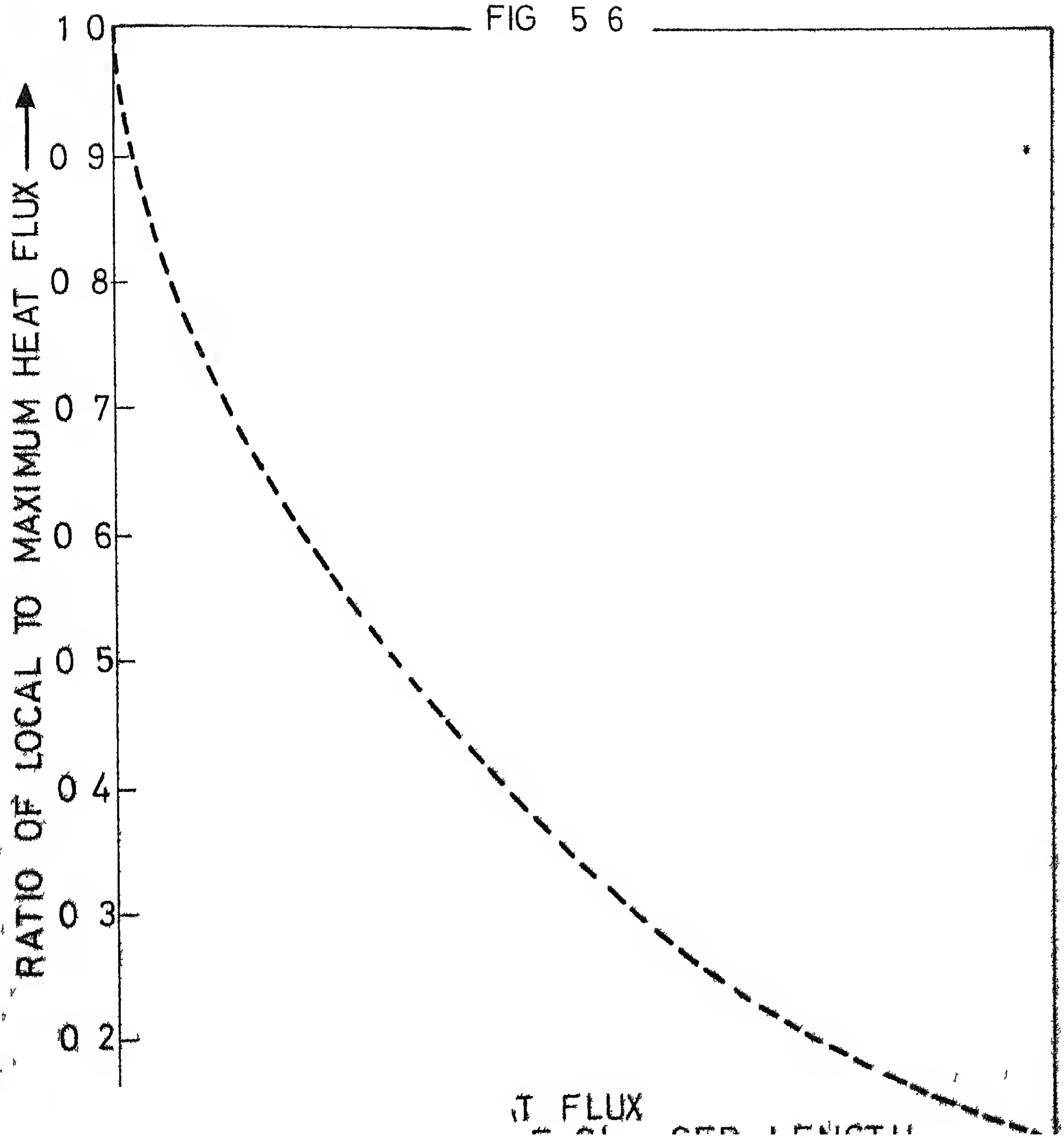

520
480
440
400
380
MANUFACTURE'S DATA
WITH K APPROXIMATION
WITHOUT K APPROXIMATION
387
8
MANUFACTURER S' DATA
TEMPERATURE (°C)
TEMPERATURE VARIATION ALONG

FIG 5 6


RATIO OF LOCAL TO MAXIMUM HEAT FLUX
1 0
0 9
0 8
0 7
0 6
0 5
0 4
0 3
0 2
T FLUX

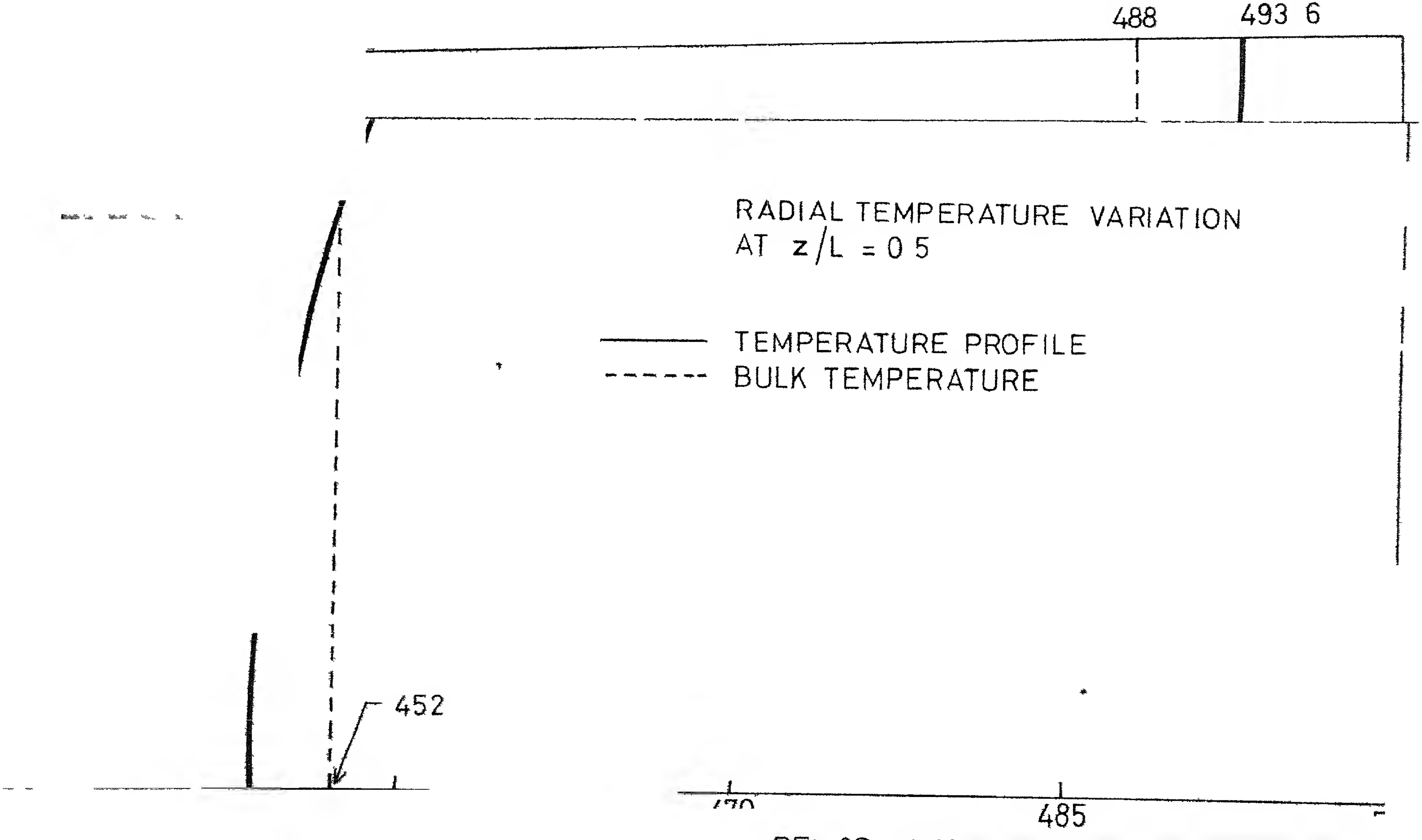
$3^{o}C$ and over prediction of secondary exit temperature sodium by $4^{o}C$ The Fig (5 5) shows predicted temperature profiles along heat exchanger length for the two cases, viz , with k_1^+ and without k_1^+ approximation The peclet numbers for the present heat exchanger are not very for removed from 100 Therefore, the effect of eddy conduction incorporated using k_1^+ is not very pronounced The two temperature profiles are therefore very close to each other except at the ends where maximum difference is observed

[5 6] VARIATION OF HEAT FLUX ALONG HEAT EXCHANGER LENGTH.

The heat flux for the present heat exchanger varies strongly along length From Fig. (5.6) we see that the heat flux at one end is ten times at the other end Hence it is incorrect to assume uniform heat flux boundary condition for present heat exchanger But results obtained for the prese case even assuming UHF do not lead to serious errors This is because for $H > 1$ the over-predicted value of efficiency is near unity and quoted value is also very close to unity

For short heat exchangers with smaller efficiency and $H \neq 1$, UHF assumption may lead to gross over prediction or under prediction of the heat exchanger performance But when $H = 1$, the present method may yield results which may be very close to UHF predictions

FIG-57


488
493 6
RADIAL TEMPERATURE VARIATION
AT z/L = 0 5
TEMPERATURE PROFILE
BULK TEMPERATURE
452
485

[5 7] RADIAL TEMPERATURE VARIATION

UHF method can predict only bulk temperature variatio of the heat exchanger along the length But present method was evolved solving fundamental equations for energy balance The solution thus gives temperature at all points within the considered boundaries Hence present method provides more information than UHF method

A detailed knowledge of temperature at all points, is of interest to the designer The radial temperature profile gives the magnitude and location of hot spots The Fig (5 7) shows that at the centre of heat exchanger, maximum temperature occurs at the outer boundary of the annulus The present method can be used to determine such radial temperature profiles, at any distances from either ends.

Only first term is required to calculate the radial temperature profile in fully developed region, i e , in the middle of heat exchanger. But close to the entrance and exit more number of terms are required Number of terms depend upon the accuracy with which temperature is to be predicted For short heat exchangers fully developed region may not be realized at all This requires large number of terms to calculate radial variation along the length

From Fig (5 7), it is found that the temperature drop in tube metal is maximum, i e , 17°C Temperature drops in primary sodium is 11°C and in secondary sodium it is 16 5°C Temperature drop is proportional to the resistance offered by the medium From Fig (5 7), we conclude that for the present case resistances in all three regions are of the same order of magnitude

[5.8] SUGGESTED FUTURE WORK

a) Recent eddy diffusivity and velocity distribution correlations for liquid metals can be made use of in determining k_1^+ accurately It was not required for the present problem because the Peclect numbers were not much greater than 100

b) The present method involves use of plug flow model where we assumed $f_1(\hat{r}_1) = g_1(\hat{r}_1) = 1$ This assumption can be removed if experimental correlations available for flow along rod bundles and flow inside tube are employed to determine $f_1(\hat{r}_1)$ and $g_1(\hat{r}_1)$ The basic energy equation can then be solved numerically as a two point boundary value problem Such a general solution will be of immense value It can be also used to ascertain validity of experimental correlations and validity of any approximate theoretical approach like equivalent annulus approach and NTU method

BIBLIOGRAPHY

1 Bailey, R V , 1950, Heat transfer to liquid metals in concentric annuli, ORNL-531

2 Bender, D J and Magee, P M , 1969, Heat transfer in rod bundles with liquid metal coolants, GEAP-10052

3 Brown, W J , 1968, Heat transfer to liquid metals in double pipe heat exchangers, ANL-7289

4 Deissler, R G and Taylor, M F , 1957, Analysis of axial turbulent flow and heat transfer through bank of rods or tubes, TID-7529 (Pt 1), Book 2, 416

5 Dwyer, O E and Tu, P S , 1960, Analytical study of heat transfer rates for parallel flow of liquid metals through tubes bundles, J Heat Transfer, 86

6 Friedland, A J and Bonilla, C F , 1961, Analytical Study of heat transfer rates for parallel flow of liquid metals through tube bundles, A I Ch E J , 7

7 Lyon, R N (ed), 1955, Liquid Metals Hand Book, US Atomic Energy Commission

8 Millsaps, K and Pohlhausen, K., 1956, Proceedings of the conference on differential equations, Univ of Maryland, Maryland

9 Nijsing, R , and Eifler, W , 1973, Temperature fields in Liquid metal cooled rod assemblies, Progress in Heat and Mass Transfer, Vol 7

10 Oberjohn, W J , 1970, Turbulent flow thermal-hydraulic characteristics of hexagonal pitch fuel assemblies, Liquid metal heat transfer and fluid dynamics, ASME

11 Raznjewic, K , 1976, Hand book of thermodynamic tables and charts, McGraw Hill

12 Schneider, P J , 1956, Heat transfer fluid mech Inst , Stanford Univ Press, Stanford, California

13 Singh, S N , 1957, Appl Sci Res A 7

14 Sleicher, C A and Tribus M , 1957, Heat transfer in a pipe with turbulent flow and arbitrary wall temperature distribution, ASME 79

15 Stein, R P , 1966, Advances in heat transfer, Vol 3

16 Kays, W M , 1975, Convective Heat and Mass Transfer, Tata McGraw Hill

17 Yevick, J G , 1966, Fast Reactor Technology Plant Design, MIT Press

APPENDIX A

[A 1] SEPARATION OF VARIABLES

The solution of Eq (3 12) and (3 13) involves H, K, K_W and $\hat{L}$ as nondimensional parameters The solution requires the knowledge of $f_1(\hat{r}_1)$ and $g_1(\hat{r}_1)$ $f_1(\hat{r}_1)$ is determined both by Prandtl and duct Reynolds number Values of these nondimensional parameters determine for a specific heat exchanger, what may be called as operating conditions for gheat exchanger

To solve Eq (3 12) and (3 13) separation of variable method is adopted Let,

$$\hat{T}_1(\hat{r}_1, \hat{z}) = \hat{E}_1(\hat{r}_1)\,\hat{\theta}(\hat{z}) \qquad (A\ 1)$$

substituting Eq (A 1) in the Eq (3.12) and (3.13) results in Eq (A 2) and Eq (A 3)

<u>Tube</u>

$$\frac{1}{\hat{\theta}(\hat{z})}\,\frac{d\hat{\theta}(\hat{z})}{d\hat{z}} = \frac{1}{g_1(\hat{r}_1)\,\hat{E}_1(\hat{r}_1)\,\hat{r}_1}\,\frac{d}{d\hat{r}_1}\left[\,f_1(\hat{r}_1)\hat{r}_1\,\frac{d\hat{E}_1(\hat{r}_1)}{d\hat{r}_1}\right] \qquad (A\ 2)$$

<u>Annulus</u>

$$\frac{1}{\hat{\theta}(\hat{z})}\,\frac{d\hat{\theta}(\hat{z})}{d\hat{z}} = \frac{-1}{g_2(\hat{r}_2)\,\omega^2\,\hat{E}_2(\hat{r}_2)(\hat{r}_2+\sigma)}\,\frac{d}{d\hat{r}_2}\left[(\hat{r}_2+\sigma)\,f_2(\hat{r}_2)\,\frac{d\hat{E}_2(\hat{r}_2)}{d\hat{r}_2}\right] \qquad (A\ 3)$$

The left hand sides of the above equations are functions of $\hat{z}$ only whereas the right hard sides are functions of $\hat{r}_1$ only , consequently, they can be equal only if they each equal a constant, $-\lambda^2$

$$\frac{1}{\hat{\theta}(\hat{z})} \frac{d\hat{\theta}(\hat{z})}{d(\hat{z})} = -\lambda^2 \tag{A 4}$$

or $$\hat{\theta}(\hat{z}) = e^{-\lambda^2 \hat{z}} \tag{A 5}$$

The right hand side of Eq (A 2) and (A 3) can be

$$\frac{d}{d\hat{r}_1}\left[f_1(\hat{r}_1)\,\hat{r}_1 \frac{d\hat{E}_1(\hat{r}_1)}{d\hat{r}_1}\right] + \lambda^2 g_1(\hat{r}_1)\,\hat{r}_1\,\hat{E}_1(\hat{r}_1) = 0 \tag{A.6}$$

and

$$\frac{d}{d\hat{r}_2}\left[f_2(\hat{r}_2)(\hat{r}_2+\sigma) \frac{d\hat{E}_2(\hat{r}_2)}{d\hat{r}_2}\right] - \lambda^2\omega^2 g_2(\hat{r}_2)\,(\hat{r}_2+\sigma)\hat{E}_2(\hat{r}_2) = \tag{A 7}$$

The interior boundary conditions become,

$$\frac{d\hat{E}_1(0)}{d\hat{r}_1} = 0 \tag{A.8}$$

$$K \frac{d\hat{E}_1(1)}{d\hat{r}_1} = \frac{d\hat{E}_2(0)}{d\hat{r}_2} \tag{A.9}$$

$$K_w \frac{d\hat{E}_1(1)}{d\hat{r}_1} = \hat{E}_2(0) - \hat{E}_1(1) \tag{A 10}$$

$$\frac{d\hat{E}_2(1)}{d\hat{r}_2} = 0 \qquad \text{(A 11)}$$

[A 2] TWO REGION STURM-LIOUVILLE PROBLEM

The Eq (A 6) and (A 7) correspond to what may be called a two region Sturm-Liouville problem and may be written as a single differential equation

$$\frac{d}{dx}\left(k \frac{dy}{dx}\right) + \lambda^2 gy = 0 , \qquad 0 \leq x \leq 2 \qquad \text{(A 12)}$$

with boundary conditions,

$$y'(0) = 0 \text{ and } y'(2) = 0 \qquad \text{(A.13)}$$

where $x = 0$ and $x = 2$ correspond to $\hat{r}_1 = 0$ and $\hat{r}_2 = 1$ respectively

The functions y, k, g are defined over both regions, tube and annulus, of the heat exchanger by,

<u>Tube</u>

$$(x = \hat{r}_1 ; \quad 0 \leq \hat{r}_1 \leq 1)$$

$$y = \hat{E}_1(\hat{r}_1) \qquad \text{(A 14)}$$

$$k = \hat{r}_1 \, f_1(\hat{r}_1) \qquad \text{(A 15)}$$

$$g = \hat{r}_1 \, g_1(\hat{r}_1) \qquad \text{(A 16)}$$

Annulus $\quad (x = \hat{r}_2 + 1 , \quad 0 \leq \hat{r}_2 \leq 1)$

$$y = \hat{E}_2(\hat{r}_2) \tag{A 17}$$

$$k = (\hat{r}_2 + \sigma)\, f_2\, (\hat{r}_2) \tag{A 18}$$

$$g = -\,\omega^2\, (\hat{r}_2 + \sigma)\, g_2(\hat{r}_2) \tag{A 19}$$

where the function y must meet the compatibility conditions at the inner wall of heat exchanger Eq (A 9) and (A 10) and the functions k and g have finite discontinuities at the inner wall of heat exchanger

The difficulty in applying the Sturm-Liouville theory to the present system lies in the fact that it specifies that the functions must be continuous The functions are not continuous in the system being considered Because of the fact that the functions are continuous in their regions, i e , piece-wise continuous, Stein (1966) has been able to show the Sturm-Liouville theory applies to the present system

According to Stein, then, there are infinite sets of eigenvalues and corresponding eigenfunctions, having limit points from $-\infty$ to ∞ Thus in one region problem the negative eigenvalues do not exist but for two region problem, having discontinuity at interface, both negative and positive eigenvalues exist

The Eq (A 4), (A 6) and (A.7) according to above two region Sturm-Liouville problem have infinite sets of positive and negative eigenvalues and corresponding eigenfunctions These equations thus may be written as,

$$\frac{1}{\hat{\theta}(\hat{z})} \frac{d\hat{\theta}(\hat{z})}{d\hat{z}} = -\lambda_n^2 \qquad (A\ 20)$$

$$\text{or} \quad \theta(z) = e^{\lambda_n^2 \hat{z}} \qquad (A\ 21)$$

$$\frac{d}{d\hat{r}_1}\left[f_1(\hat{r}_1)\ \hat{r}_1 \frac{d\hat{E}_{1,n}(\hat{r}_1)}{d\hat{r}_1}\right] + \lambda_n^2\ \hat{r}_1\ g_1(\hat{r}_1)\ E_{1,n}(\hat{r}_1) = 0$$

$$(A\ 22)$$

$$\frac{d}{d\hat{r}_2}\left[f_2\ (\hat{r}_2)\ (\hat{r}_2+\sigma) \frac{d\hat{E}_{2,n}(\hat{r}_2)}{d\hat{r}_2}\right]$$

$$- \lambda_n^2\ \omega^2\ (\hat{r}_2+\sigma)\ g_2(\hat{r}_2)\ \hat{E}_{2,n}(\hat{r}_2) = 0 \qquad (A\ 23)$$

and boundary conditions,

$$\frac{d\hat{E}_{1,n}(0)}{d\hat{r}_1} = 0 \qquad (A\ 24)$$

$$K \frac{d\hat{E}_{1,n}(0)}{d\hat{r}_1} = 0\ \frac{d\hat{E}_{2,n}(0)}{d\hat{r}_2} \qquad (A\ 25)$$

$$K_w \frac{d\hat{E}_{1,n}(1)}{d\hat{r}_1} = \hat{E}_{2,n}(0) - \hat{E}_{1,n}(1) \qquad (A\ 26)$$

$$\frac{d\hat{E}_{2,n}(1)}{d\hat{r}_2} = 0 \qquad (A.27)$$

$$n = \pm 0, \pm 1, \pm 2, \quad , \pm \infty$$

where $\hat{E}_{1,n}$ is the eigenfunction corresponding to λ_n^2

[A 3] GENERAL SOLUTION

The general solution to the Eq (A 6) and (A 7) may be written for region 1 and 2 as follows

Tube

$$\hat{T}_1(\hat{r}_1,\hat{z}) = \sum_{n=-\infty}^{n=+\infty} \hat{C}_n \hat{E}_{1,n}(\hat{r}_1)\, e^{-\lambda_n^2 \hat{z}} \qquad (A\ 28)$$

Annulus

$$\hat{T}_2(\hat{r}_2,\hat{z}) = \sum_{n=-\infty}^{n=+\infty} \hat{C}_n \hat{E}_{2,n}(\hat{r}_2)\, e^{-\lambda_n^2 \hat{z}} \qquad (A\ 29)$$

where $\hat{C}_n$ = Expansion coefficient associated with n-th eigenvalue

In order to determine eigenvalues, eigenfunctions and expansion coefficients we first of all derive orthogonality property

[A 4] ORTHOGONALITY OF EIGENFUNCTIONS

The orthogonality property of eigenfunctions may be derived in the following way Eq (A.22) is first multiplied by $\hat{E}_{1,m}$ to obtain a relation between different order eigenfunctions A second relation is obtained by interchanging m and n in first relation. The two expressions are then integrated between 0 and 1, subtracted and simplified to yield Eq (A 30) below

$$(\lambda_n^{\,2} - \lambda_m^{\,2}) \;\; B_1 \;\{\hat{E}_{1,n}\;(\hat{r}_1)\;\hat{E}_{1,m}\;(\hat{r}_1)\}$$

$$= \;\; 2\hat{E}_{1,n}(1) \;\; [\; \frac{d\hat{E}_{1,m}\;(1)}{d\hat{r}_1} - \hat{E}_{1,m}(1)\;\frac{d\hat{E}_{1,n}(1)}{d\hat{r}_1}] \qquad (A\ 30)$$

A similar treatment to Eq (A 23) yields Eq (A 31) given below

$$(\lambda_n^{\,2} - \lambda_m^{\,2}) \; HB_2 \;\{\hat{E}_{2,n}(\hat{r}_2)\;\hat{E}_{2,m}(\hat{r}_2)\}$$

$$= \;\; \frac{2}{K}\;[\hat{L}_{2,m}(0)\;\frac{d\hat{E}_{2,n}(0)}{d\hat{r}_2} - \hat{E}_{2,n}(0)\;\frac{d\hat{E}_{2,n}\;(0)}{d\hat{r}_2}] \qquad (A\ 31)$$

where $n \;=\; \pm\,0,\; \pm\,1,\; \pm\,2,$

The Eq (A.30) and (A 31) are combined by applying the innerwall boundary conditions to yield

$$(\lambda_n^{\,2} - \lambda_m^{\,2})\;[B_1\;\{\hat{E}_{1,n}(\hat{r}_1)\;\hat{E}_{1,m}(\hat{r}_1)\}$$

$$- \; HB_2\;\{\hat{E}_{2,n}(\hat{r}_2)\;\hat{E}_{2,m}(\hat{r}_2)\}] \;\; = \;\; 0 \qquad (A\ 32)$$

$$\begin{matrix} m \\ n \end{matrix} \;=\; \pm\,0,\; \pm\,1,\; \pm\,2,$$

This gives desired orthogonality condition

$$B_1\;\{\hat{E}_{1,n}(\hat{r}_1)\;\hat{E}_{1,m}(\hat{r}_1)\} - \;HB_2\;\{\hat{E}_{2,n}\;(\hat{r}_2)\;\hat{E}_{2,m}(\hat{r}_2)\} \;=\; 0;$$

$$n \neq m \qquad (A\ 33)$$

$$B_1\{[E_{1,n}(\hat{r}_1)]^2\} - HB_2\{[E_{2n}(\hat{r}_2]^2\} = N_n \quad ; \quad n = m \tag{A 34}$$

$$n = \pm 0; \ \pm 1, \ \pm 2 ,$$

where $B_1\{\ \}$ and $B_2\{\ \}$ physically mean average over cross-section They are defined as follows

Tube

$$B_1\{\ \} = 2\int_0^1 g_1(\hat{r}_1)\{\ \}\ \hat{r}_1\ d\hat{r}_1 \tag{A 35}$$

Annulus

$$B_2\{\ \} = \frac{2}{1 + 2\sigma} \int_0^1 g_2(\hat{r}_2)\{\ \}(\hat{r}_2 + \sigma)\ d\hat{r}_2 \tag{A.36}$$

Thus the average temperature over the cross-section (bulk average) is given as $B_1\{\hat{T}_1(\hat{r}_1, \hat{z})\}$, which is denoted as $\hat{T}_{1av}(\hat{z})$

Note that dimensional equivalent for bulk averages are defined for tube and annulus, as follows

Tube

$$B_1\{\ \} = \left(2\pi \int_0^{r_{12}} u_1(r_1)\ r_1\ \{\ \}dr_1\right)/\left(2\pi \int_0^{r_{12}} u_1(r_1) r_1\ dr_1\right) \tag{A 37}$$

Annulus

$$B_2\{\ \} = \left(2\pi \int_{r_{21}}^{r_{22}} u_2(r_2)(r_2)\{\ \}\, dr_2 \right) / \left(2\pi \int_{r_{21}}^{r_{22}} u_2(r_2) r_2\, dr_2 \right)$$

(A 38)

[A 5] EIGENFUNCTIONS

Solution of the Eq (A 22) and (A 23) for $\lambda_o^2 = 0$ are $E_{1,0}$ and $E_{2,0}$ both equal to a constant No generality is lost in assuming these constant equal unity

$$\hat{E}_{1,0} = \hat{E}_{2,0} = 1 \qquad \text{(A 39)}$$

The zeroth order normalizing factor as given by Eq (3 61) then becomes,

$$N_o = 1-H \qquad \text{(A 40)}$$

Solution of Eq (A 22) and)(A 23) is best effected by defining auxiliary functions $F(\hat{r}_1, \lambda_n)$ and $G(\hat{r}_2, \lambda_n)$ Let

$$\hat{E}_{1,n}(\hat{r}_1) = A_n' F(\hat{r}_1, \lambda_n) \qquad \text{(A 41)}$$

$$\hat{E}_{2,n}(\hat{r}_2) = B_n' G(\hat{r}_2, \lambda_n) \qquad \text{(A 42)}$$

where $F(\hat{r}_1, \lambda_n)$ and $G(\hat{r}_2, \lambda_n)$ are solutions to

$$\frac{d}{d\hat{r}_1}\left[f_1(\hat{r}_1)\, \hat{r}_1 \frac{dF(\hat{r}_1, \lambda_n)}{d\hat{r}_1}\right] + \lambda_n^2\, \hat{r}_1\, g_1(\hat{r}_1)\, F(\hat{r}_1, \lambda_n) = 0$$

(A 43)

and,

$$\frac{d}{d\hat{r}_2}\left[f_2(\hat{r}_2)\,(\hat{r}_2+\sigma)\,\frac{dG(\hat{r}_2,\ \lambda_n)}{d\hat{r}_2}\right] -$$

$$- \lambda_n^{\,2}\,\omega^2\,g_2(\hat{r}_2)(\hat{r}_2+\sigma)\,G(\hat{r}_2,\ \lambda_n) = 0 \qquad (A\ 44)$$

Initial Conditions

$$\frac{dF(0,\ \lambda_n)}{d\hat{r}_1} = 0; \quad \frac{dF(1,\ \lambda_n)}{d\hat{r}_2} = 0\ ; \qquad (A\ 45)$$

$$F(0,\ \lambda_n) = 1\ ; \quad G(1,\ \lambda_n) = 1\ ; \qquad (A\ 46)$$

The constants are so defined that they satisfy inner wall boundary condition Eq (A 25),

$$K\ \frac{d\hat{E}_{1,n}(1)}{d\hat{r}_1} = \frac{d\hat{E}_{2,n}\ (0)}{d\hat{r}_2} \qquad (A\ 25)$$

which in terms of auxiliary functions become,

$$KA'_n\ \frac{dF(1,\ \lambda_n)}{d\hat{r}_1} = B'_n\ \frac{dG(0,\ \lambda_n)}{d\hat{r}_2} \qquad (A\ 47)$$

Therefore let,

$$A'_n = \frac{dG(0,\ \lambda_n)}{d\hat{r}_2} \qquad (A\ 48)$$

and $$B'_n = \frac{dF(1,\ \lambda_n)}{d\hat{r}_2} \qquad (A\ 49)$$

where $n = \pm 1, \pm 2, \pm 3,$ (A 50)

[A 6] EIGENVALUE EQUATION

The eigenvalues are determined by the innerwall boundary condition Eq (A 26) which in terms of auxiliary functions F and G becomes

$$K_w \frac{dG(0, \lambda_n)}{d\hat{r}_2} \frac{dF(1, \lambda_n)}{d\hat{r}_1} = K \frac{dF(1, \lambda_n)}{d\hat{r}_1} G(0, \lambda_n) - \frac{dG(0, \lambda_n)}{d\hat{r}_2} F(1, \lambda_n) \qquad (A\ 51)$$

$$n = \pm 1, \pm 2, \pm 3,$$

The eigenvalue equation is therefore defined as,

$$(\phi_n) = K_w \frac{dG(0, \lambda_n)}{d\hat{r}_2} \frac{dF(1, \lambda_n)}{d\hat{r}_1} + \frac{dG(0, \lambda_n)}{d\hat{r}_2} F(1, \lambda_n) - K \frac{dF(1, \lambda_n)}{d\hat{r}_1} G(0, \lambda_n) \qquad (A\ 52)$$

The eigenvalues λ_n^2 are obtained from the roots of the equation,

$$(\phi_n) = 0 \qquad (A.53)$$

with $\lambda_0^2 = 0$

[A 7] EXPANSION COEFFICIENTS

Temperature distributions in tube and annulus are given by Eq (A 28) and (A 29)

<u>Tube</u>

$$\hat{T}_1 (\hat{r}_1, \hat{z}) = \sum_{n=-\infty}^{n=+\infty} \hat{C}_n \hat{E}_{1,n} (\hat{r}_1) e^{-\lambda_n^2 \hat{z}} \qquad (A\ 28)$$

<u>Annulus</u>

$$\hat{T}_2(\hat{r}_2, \hat{z}) = = \sum_{h=-\infty}^{n=+\infty} \hat{C}_n \hat{E}_{2,n} (\hat{r}_2) e^{-\lambda_n^2 \hat{z}} \qquad (A\ 29)$$

Relation defining expansion coefficients, C_n, are obtained from the entrance and exit temperature profiles, of the heat exchanger fluids

$\hat{z} = 0$

$$\hat{T}_1(\hat{r}_1, 0) = 0 \qquad (A\ 54)$$

$$\hat{T}_2(\hat{r}_2, 0) = P_2(\hat{r}_2) \quad \text{(unknown)} \qquad (A\ 55)$$

$\hat{z} = \hat{L}$

$$\hat{T}_1(\hat{r}_1, L) = P_1(\hat{r}_1) \quad \text{(unknown)} \qquad (A\ 56)$$

$$\hat{T}_2(\hat{r}_2, \hat{L}) = 1 \qquad (A\ 57)$$

The end temperature distributions may be written in terms of the series solution,

$\hat{z} = 0$

$$0 = \sum_{n=-\infty}^{+\infty} \hat{C}_n \hat{E}_{1,n} (\hat{r}_1) \qquad (A\ 58)$$

$$p_2(\hat{r}_2) = \sum_{n=-\infty}^{+\infty} \hat{C}_n \hat{E}_{2,n} (\hat{r}_2) \qquad (A\ 59)$$

$\hat{z} = \hat{L}$

$$p_1(\hat{r}_1) = \sum_{n=-\infty}^{+\infty} \hat{C}_n \hat{E}_{1,n} (\hat{r}_1)\, e^{-\lambda_n^2 \hat{L}} \qquad (A\ 60)$$

$$1 = \sum_{n=-\infty}^{+\infty} \hat{C}_n \hat{E}_{2,n} (\hat{r}_2)\, e^{-\lambda_n^2 \hat{L}} \qquad (A\ 61)$$

Manipulation of above equations yields expressions for the expansion coefficients Eq (A 58) and (A 59) are multiplied by $\hat{E}_{1,n}$ and $H\hat{E}_{2,n}$, respectively, and the bulk average operation is applied to the result The expressions are then subtracted and the orthogonality condition, Eq (A 33) and (A 34) applied to give Eq. (A 62) below A similar treatment to Eq (60) and (A 61) yield Eq (A 63)

$$\hat{C}_n N_n = - HB_2 \{\hat{E}_{2,n} (\hat{r}_2)\, p_2(\hat{r}_2)\} \qquad (A.62)$$

$$\hat{C}_n N_n e^{-\lambda_n^2 \hat{L}} = B_1 \{P_1(\hat{r}_1)\, \hat{E}_{1,n}(\hat{r}_1)\} - HB_2 \{\hat{E}_{2,n} (\hat{r}_2)\} \qquad (A.63)$$

the above expressions cannot be used directly as they contain the unknown expressions $p_1(\hat{r}_1)$ and $P_2(\hat{r}_2)$ However, these unknown functions can be eliminated by substituting them as their series expansions After substitution the two Eq (A 62) and (A 63) are added to give Eq (A 64)

$$\hat{C}_n N_n (1 + e^{-\lambda_n^2 \hat{L}}) = \sum_{k=-\infty}^{+\infty} \hat{C}_k [e^{-\lambda_k^2 \hat{L}}$$

$$\times B_1 \{\hat{L}_{1,n} (\hat{r}_1) \hat{E}_{1,k} (\hat{r}_1)\} - HB_2 \{\hat{E}_{2,n}(\hat{r}_2)\hat{E}_{2,k}(\hat{r}_2)\}$$

$$- HB_2 \{\hat{E}_{2,n}(\hat{r}_2)\} \qquad (A\ 64)$$

$$n = 0, \pm 1, \pm 2, \pm 3,$$

For $n \neq 0$, the term for $K = 0$ drops out of above summation and $\hat{C}_o$ does not appear in the expression An explicit relation for $\hat{C}_o$ in terms of higher-ordered expansion coefficients may be obtained for $n = 0$

For $n = 0$, Eq (A.64) becomes,

$$\hat{C}_o N_o = \sum_{k=-\infty}^{+\infty} \hat{C}_k (e^{-\lambda_k^2 \hat{L}} - 1) B_1 \{\hat{E}_{1,k} (\hat{r}_1)\} - H \qquad (A\ 65)$$

A fairly straightforward manipulation of the Eq (A 64) yields the following simplified form,

$$\sum_{k=-\infty}^{+\infty} \hat{C}_k \left(1-e^{-\lambda_k^2 \hat{L}}\right) Q_{n,k} = -B_1 \{\hat{E}_{1,n}(\hat{r}_1)\} \qquad (A\ 66)$$

where,

$$Q_{n,k} = B_1 \{\hat{E}_{1,n}(\hat{r}_1)\, \hat{E}_{1,k}(\hat{r}_1)\} \quad ; \quad k \neq n \qquad (A\ 67)$$

$$Q_{n,n} = [B_1\{[E_{1,n}(r_1)]^2\} + \left(\frac{e^{-\lambda_k^2 \hat{L}}}{1-e^{-\lambda_1^2 \hat{L}}}\right) N_n] \; ; \; k = n$$

(A 68)

$n = \pm 1, \pm 2, \pm 3,$

The zeroth order expansion coefficient does not appear in the above equation and must be calculated by Eq (A.65)

[A 8] NORMALIZED EIGENFUNCTIONS

At this point it becomes desirable from practical viewpoint to normalize the eigenfunction with respect to the square root of the absolute value of N_n Absolute value is used because for $n < 0$, N_n becomes negative This normalization is desirable for computational considerations, since it was found that unnormalized eigenfunctions become unmanageably large as their order is increased Let

$$E_{1,n}(\hat{r}_1) \rightarrow \frac{\hat{E}_{1,n}(\hat{r}_1)}{\sqrt{|N_n|}} \qquad (A\ 69)$$

$$E_{2,n}(\hat{r}_2) \rightarrow \frac{\hat{E}_{2,n}(\hat{r}_2)}{\sqrt{|N_n|}} \qquad (A\ 70)$$

All the eigenfunctions are normalized except zeroth-order eigenfunctions, which remain,

$$E_{1,0} = E_{2,0} = 1$$

using normalized eigenfunctions, the orthogonality conditions become

$$B_1 \{E_{1,n}(\hat{r}_1)\, E_{1,m}(\hat{r}_1)\} - HB_2 \{E_{2,n}(\hat{r}_2)\, E_{2,m}(\hat{r}_2)\} = 0 \; ; \quad n \neq m \qquad (A\ 71)$$

$$B_1 \{[E_{1,n}(\hat{r}_1)]^2\} - HB_2 \{[E_{2,n}(\hat{r}_2)]^2\} = \begin{matrix} +1 & n > 0 \\ -1 & n < 0 \end{matrix} \qquad n = \qquad (A\ 72)$$

Relations which were derived so for one still valid with the value of N_n taken as + 1 for n > 0 and - 1 for n < 0

[A 9] DEFINITIONS TO AVOID USE OF NEGATIVE INDICES

Eigenvalues are given as

$$\lambda^2_{-m},\ \lambda^2_{-(m-1)},\ \ \ ,\ \lambda^2_{-1},\ \lambda^2_{0},\ \lambda^2_{1},\ \ \ \lambda^2_{m-1},\ \lambda^2_{m}$$

where $-\infty \leq m \leq +\infty$

We define auxiliary eigenvalues β_n^2

Such that

$$\lambda^2_{-n} \rightarrow -\beta_n^2 \tag{A 73}$$

Eigenfunctions associated with negative indices are also redefined as,

$$E_{1,-n}(\hat{r}_1) = E^*_{1,n}(\hat{r}_1) \tag{A 74}$$

$$E_{2,-n}(\hat{r}_2) = E^*_{2,n}(\hat{r}_2) \tag{A 75}$$

Expansion coefficients associated with the negative eigenvalues may be redefined in terms of auxiliary expansion coefficients A_n and C_n

$$\hat{C}_n \rightarrow C_n \tag{A 76}$$

$$C_{-n} \rightarrow (e^{-\beta_n^2 \hat{L}})A_n \tag{A 77}$$

The above definitions now give summation for λ_n^2 and β_n^2 from 0 to $+\infty$ and thus obviates use of negative indices

APPENDIX B

UNIFORM HEAT FLUX SAMPLE CALCULATION FOR KALPAKKAM DATA

[B 1] NUMBER OF HEAT TRANSFER UNITS

The number of heat transfer units (NTU) for a heat exchanger is defined as $\overline{U}A/C_{min}$ where $\overline{U}A$ for double pipe is defined from equation (B 1),

$$\frac{1}{\overline{U}A} = \frac{1}{2\pi k_w L}\left[\frac{2k_w}{h_1 A_1} + \frac{2k_w}{h_2 A_2} + \ln\left(\frac{r_{21}}{r_{22}}\right)\right] \qquad (B\ 1)$$

and C_{min} = Minimum heat capacity flow rate

A_1 = Surface area of tube

A_2 = Inner surface area of annulus

k_w = Conductivity of tube metal

h_1 = Heat transfer coefficient for tube side

h_2 = Heat transfer coefficient for annulus side

L = Length of heat exchanger

For double pipe heat exchanger the following convenient expression for NTU, in terms of Nusselt numbers can be derived from Eq (B 1),

$$\frac{1}{NTU} = \frac{C_{min}}{2\pi k_w L} \left[\frac{2k_w}{Nu_1 k_1} + \frac{2k_w \, (r_{22}-r_{21})}{Nu_2 \; k_2 \; r_{21}} + \ln \left(\frac{r_{21}}{r_{12}}\right) \right] \tag{B 2}$$

The Nusselt numbers were obtained using experimental correlations of Loyn (1955) for tube and Bailey (1950) for annulus

Tube

$$Nu_1 = 7 + 0\ 025\ Pe_1^{0\ 8} \tag{B 3}$$

Annulus

$$Nu_2 = 5\ 25 + 0\ 0188\ Pe_2^{0\ 8} \left(\frac{r_{21}}{r_{22}}\right)^{0\ 3} \tag{B 4}$$

In Eq (B 2) various parameters for Kalpakkam heat exchanger have following values

$Nu_1 = 8\ 44$

$Nu_2 = 6\ 676$

$k_w = 19\ 77$ W/m°C (at 425°C)

$k_1 = k_2 = 80$ watts/m°C)(at 425°C)

$C_{min} = 110$ watts

$L = 2\ 48$ m

$r_{21} = 11 \times 10^{-3}$ m

$r_{12} = 7 \times 10^{-3}$ m

and $2(r_{22}-r_{21})$ = Hydraulic diameter

$$\left(\frac{4 \times \text{Cross-sectional area of Annulus}}{\text{Wetted Perimeter}} \right)$$

With these values Eq (B 2) gives

$$\underline{NTU = 10\ 470}$$

[B 2] EFFICIENCY

Efficiency for a heat exchanger is defined as the ratio of actual heat transferred to that which would be transferred if heat exchanger were infinitely long For counter flow heat exchanger, efficiency interms of NTU and H $\left(H = \dfrac{\text{Primary Heat Capacity Flow Rate}}{\text{Secondary Heat Capacity Flow Rate}}\right)$ is given as

$H < 1$

$$\varepsilon = \frac{1 - \text{Exp}\ (-NTU\ (1 - H))}{1 - H\ \text{Exp}\ (-NTU\ (1-H))} \qquad \text{(B 5)}$$

$H > 1$

$$\varepsilon = \frac{1 - \text{Exp}\ (-\ NTU\ (1 - \frac{1}{H}))}{1 - \frac{1}{H}\ \text{Exp}\ (-NTU\ (1 - \frac{1}{H}))} \qquad \text{(B 6)}$$

From equation (B 6) efficiency can be calculated substituting the values of NTU and H For present case H = 1 65 Therefore Eq (B 6) gives

$$\underline{\varepsilon = 0\ 993}$$

[B 3] OUTLET TEMPERATURES

Efficiency for a given heat exchanger in terms of temperatures is defined as follows

$H > 1$

$$\varepsilon = \frac{T_{1L} - T_{10}}{T_{2L} - T_{10}} \qquad \text{(B 7)}$$

$H < 1$

$$\varepsilon = \frac{T_{2L} - T_{20}}{T_{2L} - T_{10}} \qquad \text{(B 8)}$$

where subscript

1 denotes tube side and 2 denotes annulus side,

0 denotes entrance side and L denotes exit side

From general heat balance we can write,

$$(T_{1L} - T_{10}) = H\,(T_{2L} - T_{20}) \qquad \text{(B 9)}$$

For present case secondary outlet temperature, T_{1L}, can be determined from Eq (B 7) Primary outlet temperature, T_{20}, can be determined from Eq (B 9), They are given as,

$$\underline{T_{1L} = 518^{\circ}C}$$

$$\underline{T_{20} = 377^{\circ}C}$$